A Glimpse of Representing Stochastic Processes

Nathaniel Osgood

CMPT 858

November 22, 2012

Dynamic Uncertainty: Stochastic Processes

- Examples of things commonly stochastically approximated
  - Stock market
  - Rainfall
  - Oil prices
  - Economic growth
- What considered “stochastic” will depend on the scope of the model
  - Detailed model: Individual behaviour, transmission, differential severity of infection, etc.
  - A meteorological model may not consider rainfall stochastic

Stochastic Processes in AnyLogic

- In AnyLogic, ABM and Discrete Event Models ("Network-Based Modeling") are typically stochastic
  - Transitions between states
  - Event firing
  - Messages
    - (Frequent) timing of message send
    - Target of messages
  - Duration of a procedure
- As a result, there will be variation in the results from simulation to simulation

Summarizing Variability

- To gain confidence in model results, typically need to run a "Monte Carlo" ensemble of realizations
  - Deal with means, standard deviations, and empirical fractiles
  - As is seen here, there are typically still broad regularities between most runs (e.g. rise & fall)
- Need to reason over a population of realizations $\Rightarrow$ statistics are very valuable
  - Fractile within which historic value falls
  - Mean difference in results between interventions

Monte Carlo Methods in AnyLogic

- Monte Carlo methods draw repeated samples from distributions & stochastic processes of interest
- When running Monte Carlo method, we'd like to summarize the results of multiple runs
- One option would be to display each trajectory over time; downside: quickly gets messy
- AnyLogic's solution
  - Accumulate data regarding how many trajectories fall within given areas of value for a given interval of time using a "Histogram2D Data"
  - Display the Histogram2D Chart

MonteCarlo2D Histogram

- Divides up time into user-specified # of intervals
  - This forms a set of divisions along the horizontal (time) axis
- Divides up value axis for quantity being displayed into a user-specified # of interval
  - This forms a set of divisions along the vertical (value) axis
- Together, the divisions define a uniform (2D) grid
  - For each cell on that grid, a "Histogram2D Data" object accumulates data regarding how many trajectories include a value within that cell
    - i.e. how many trajectories have hold a range of values during a given interval of time)

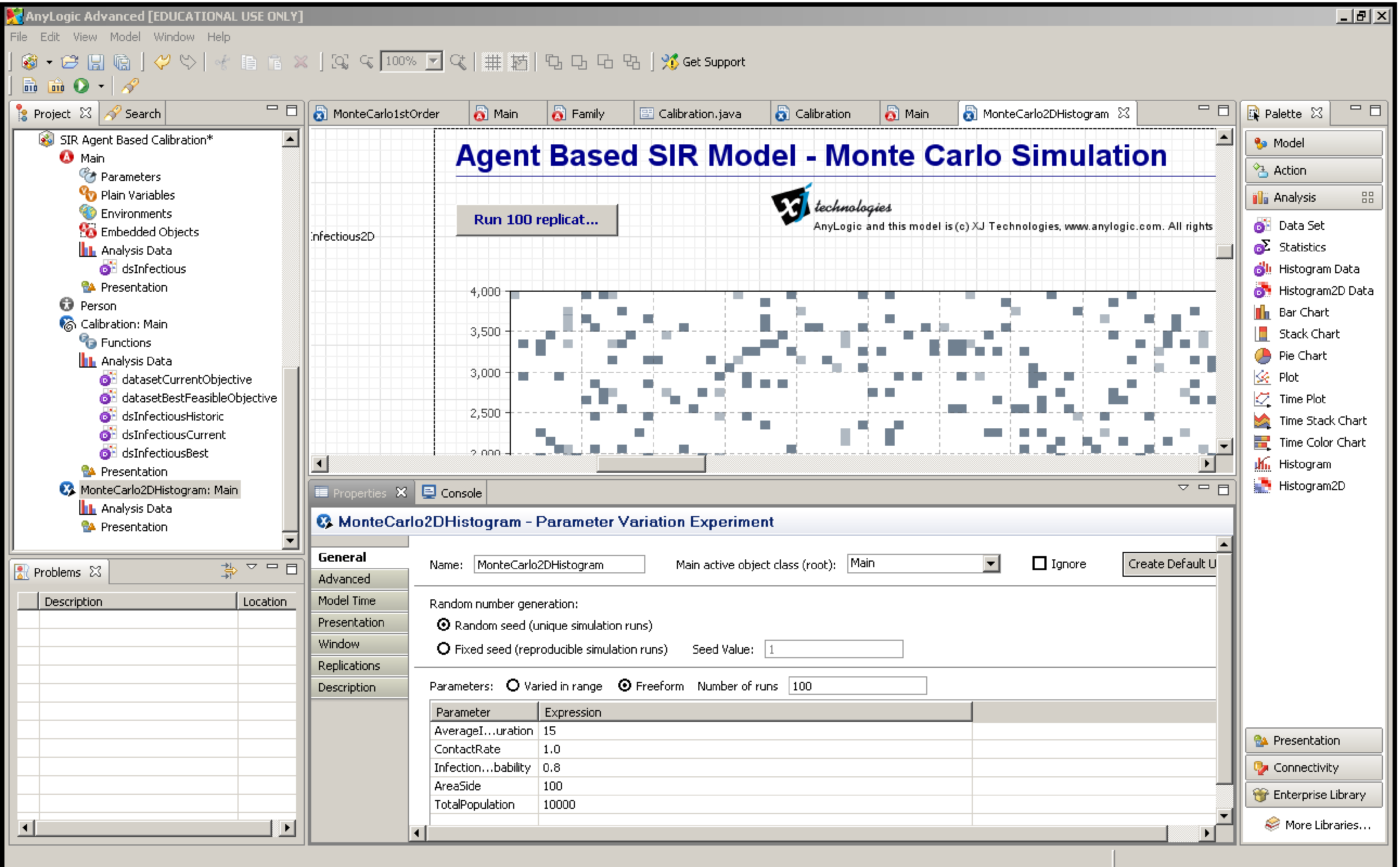
Hands on Model Use Ahead

Load Sample Model:

SIR Agent Based Calibration

(Via “Sample Models” under “Help” Menu)

Monte Carlo Analysis with Fixed Parameter Values



Results of Monte Carlo Simulation

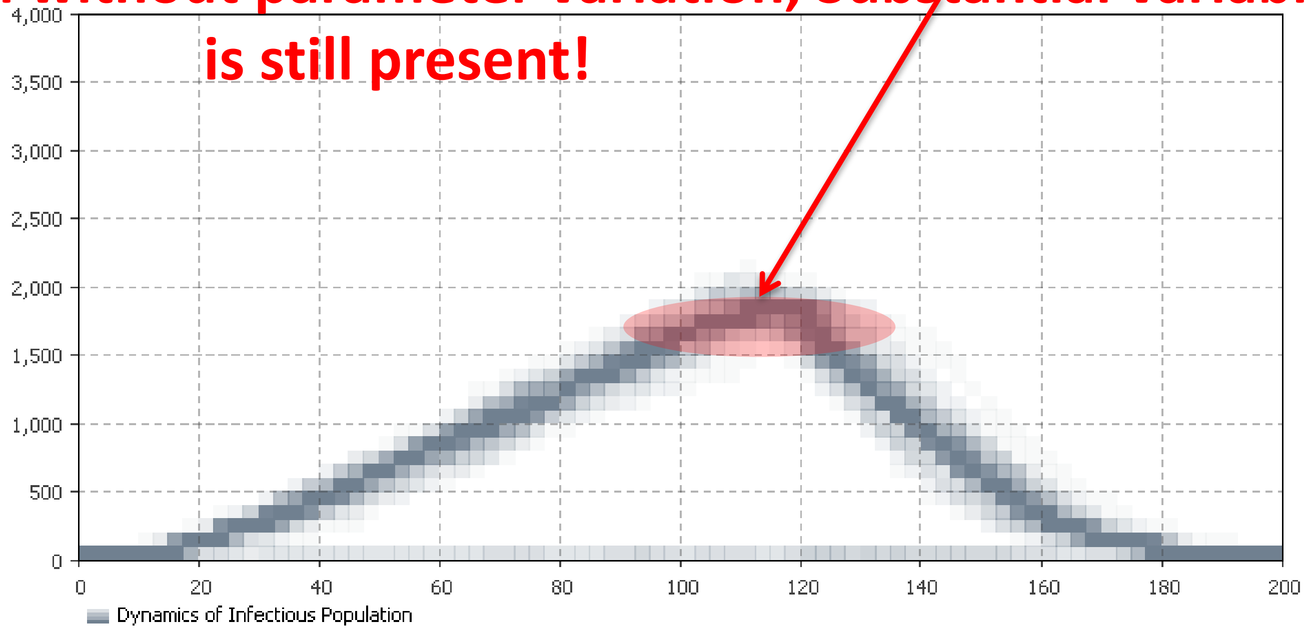
AnyLogic

Agent Based SIR Model - Monte Carlo Simulation

Run 100 replications

technologies

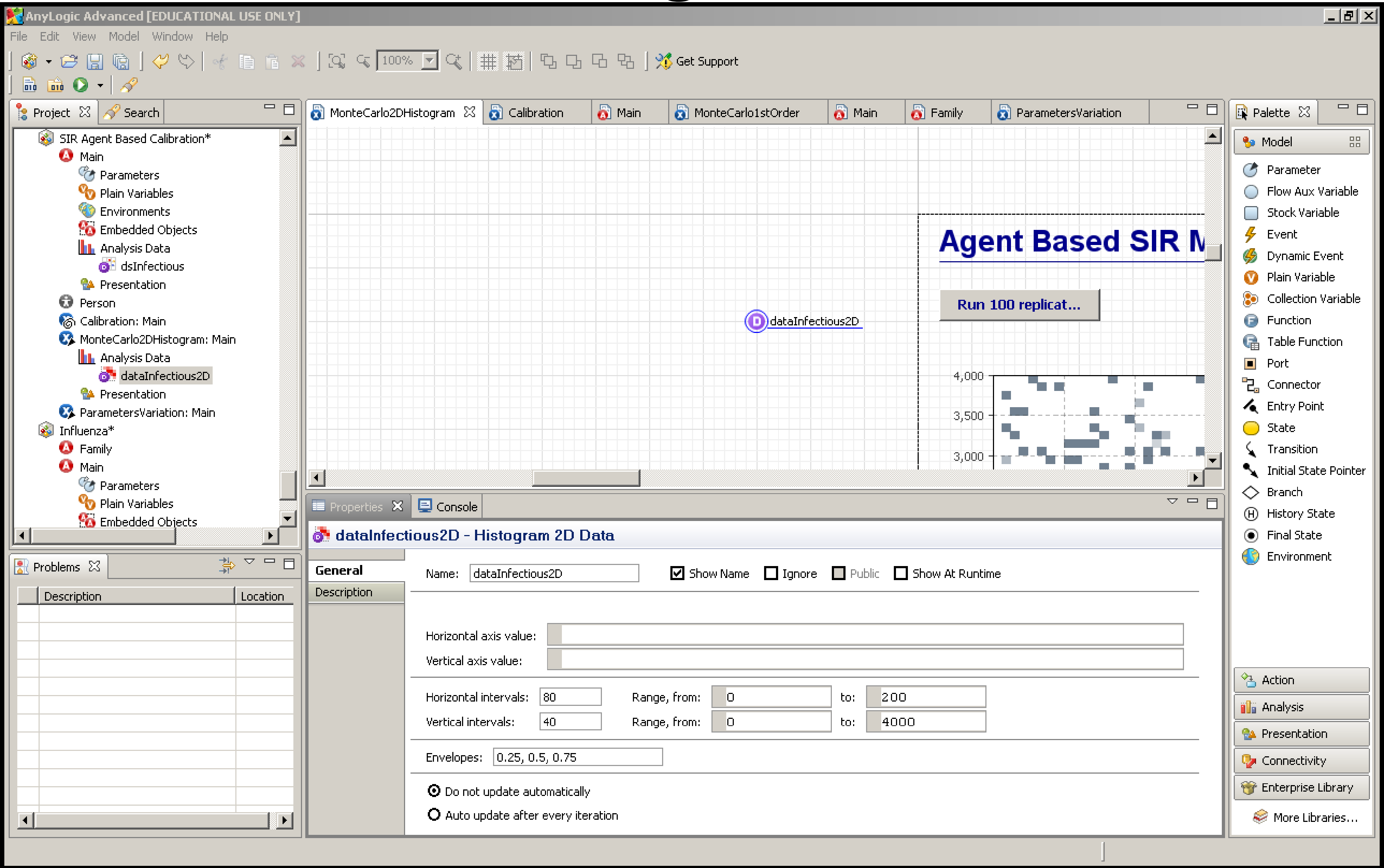
Even without parameter variation, Substantial variability is still present!



This experiment performs multiple (100) runs of the Agent Based SIR Model with SAME (default) parameter values.
As the model is essentially stochastic, each run results in a different output. In the chart above we display the summary of simulation runs (namely, the dynamics of the Infectious population size) in the form of the 2D histogram. The color intensity of a chart spot corresponds to the size of the corresponding 2D histogram bin.

Run: 100 Finished | **Experiment:** 100% | **Simulation:** Stop time not set | **Memory:** 14M of 63M | 55.1 sec

2D Histogram Data



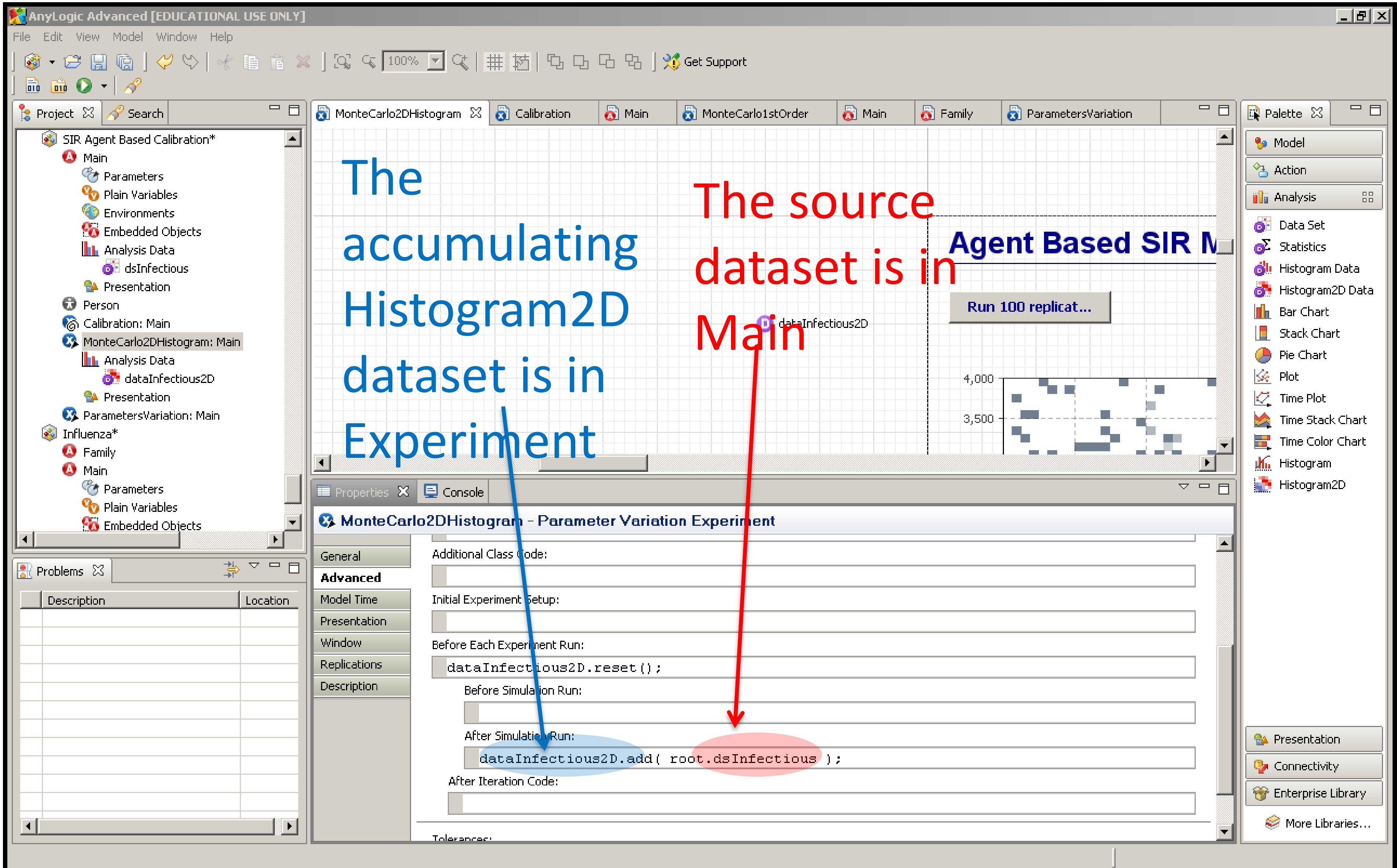
Important Distinction (Declining Order of Aggregation)

- Experiment
  - Collection of simulations
- Simulation
  - Collection of replications that can yield findings across set of replications (e.g. mean value)
- Replication
  - One run of the model

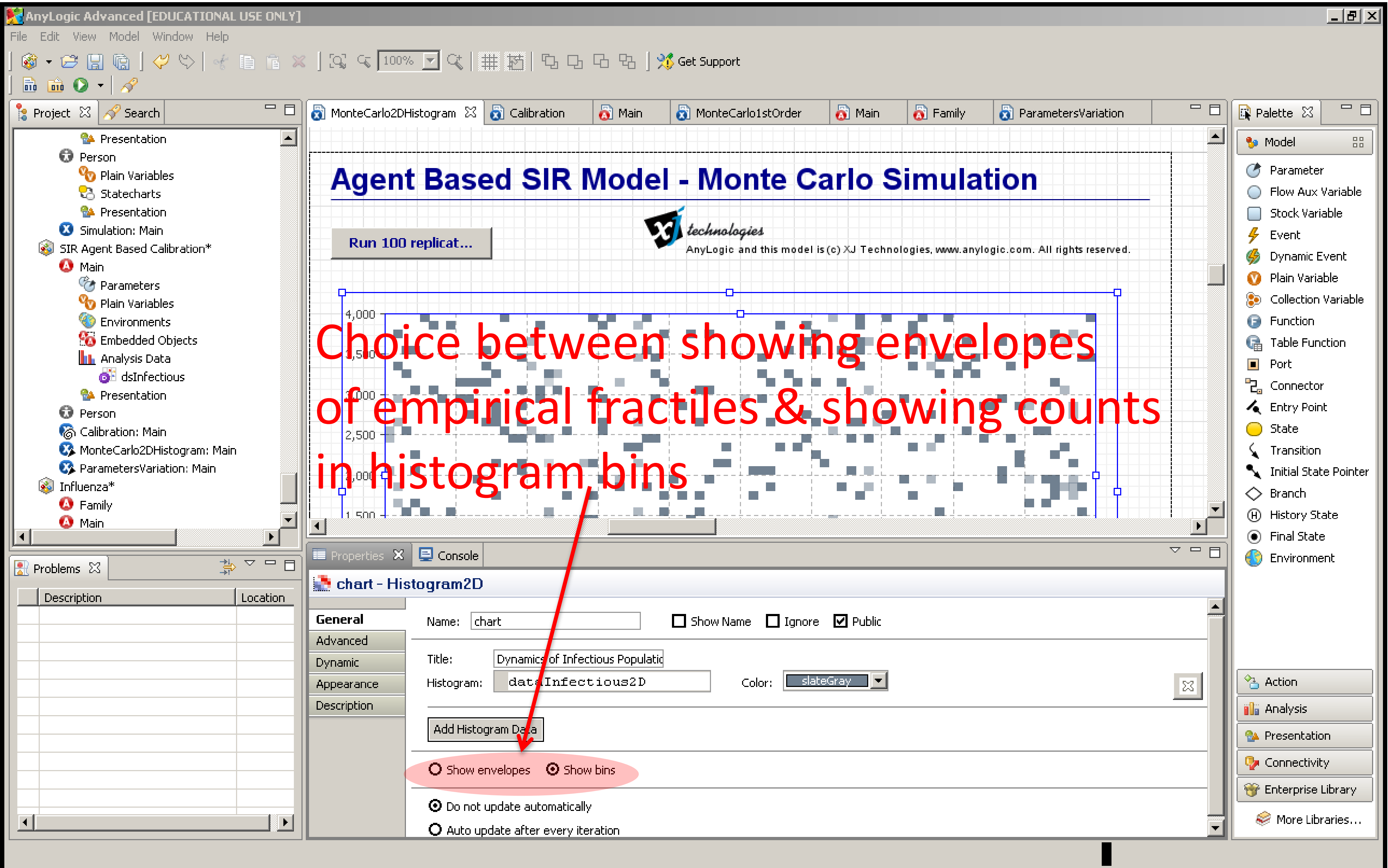
Flexibility Typically Ignored

- In most AnyLogic models, an Experiment is composed of a single Simulation, which is composed of a single Replication
- In most AnyLogic models which run "ensembles" of realizations, a simulation is composed of only a single realization

Accumulating the Histogram2D dataset from other datasets



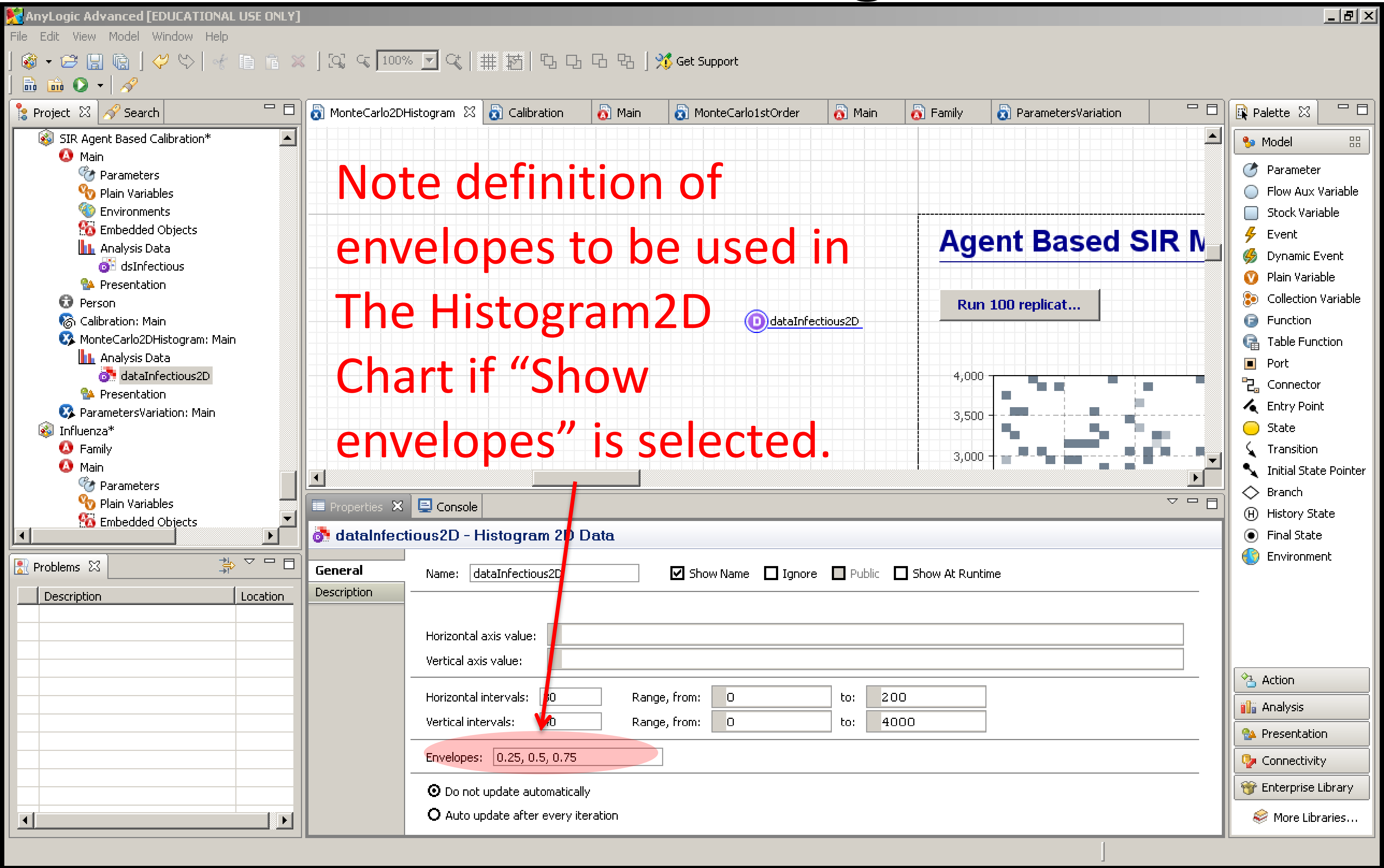
Monte Carlo Sensitivity Analyses in AnyLogic



Difference Between Chart Options "Show envelopes"

- This option shows envelopes of empirical fractiles
  - These are associated with empirical fractiles defined in terms of percentages (e.g. "0.25" means boundary between lowest and 2nd lowest quartile; "0.50" means median)
  - e.g. These define envelopes of (contours) around the median within which data from different % of realizations fall
  - A "slice" through the output at a particular moment in time would be like an **extended boxplot** (showing fractiles)
- The empirical fractiles to use are themselves defined in the associated Histogram2D Data object

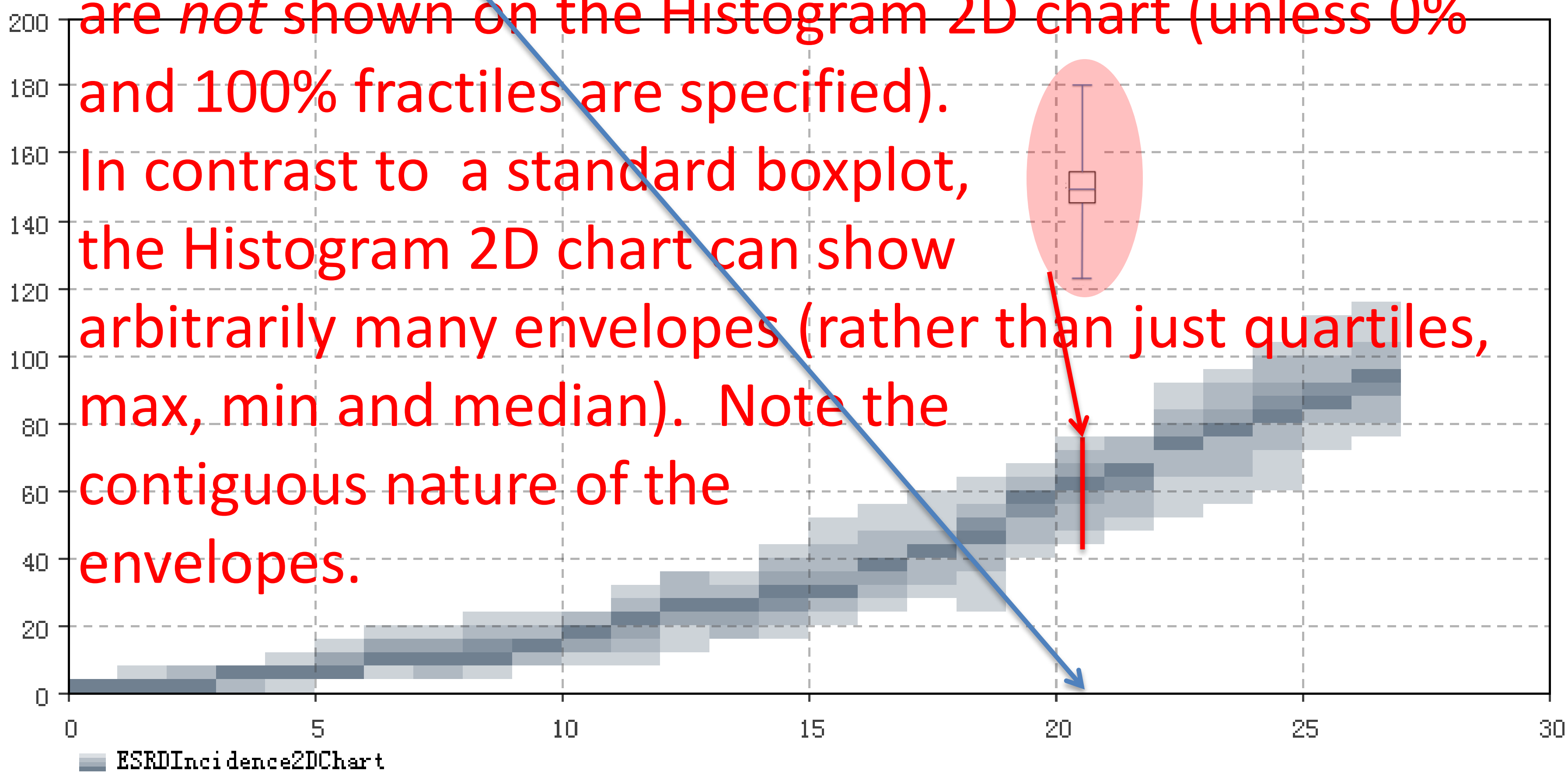
Reminder: 2D Histogram Data



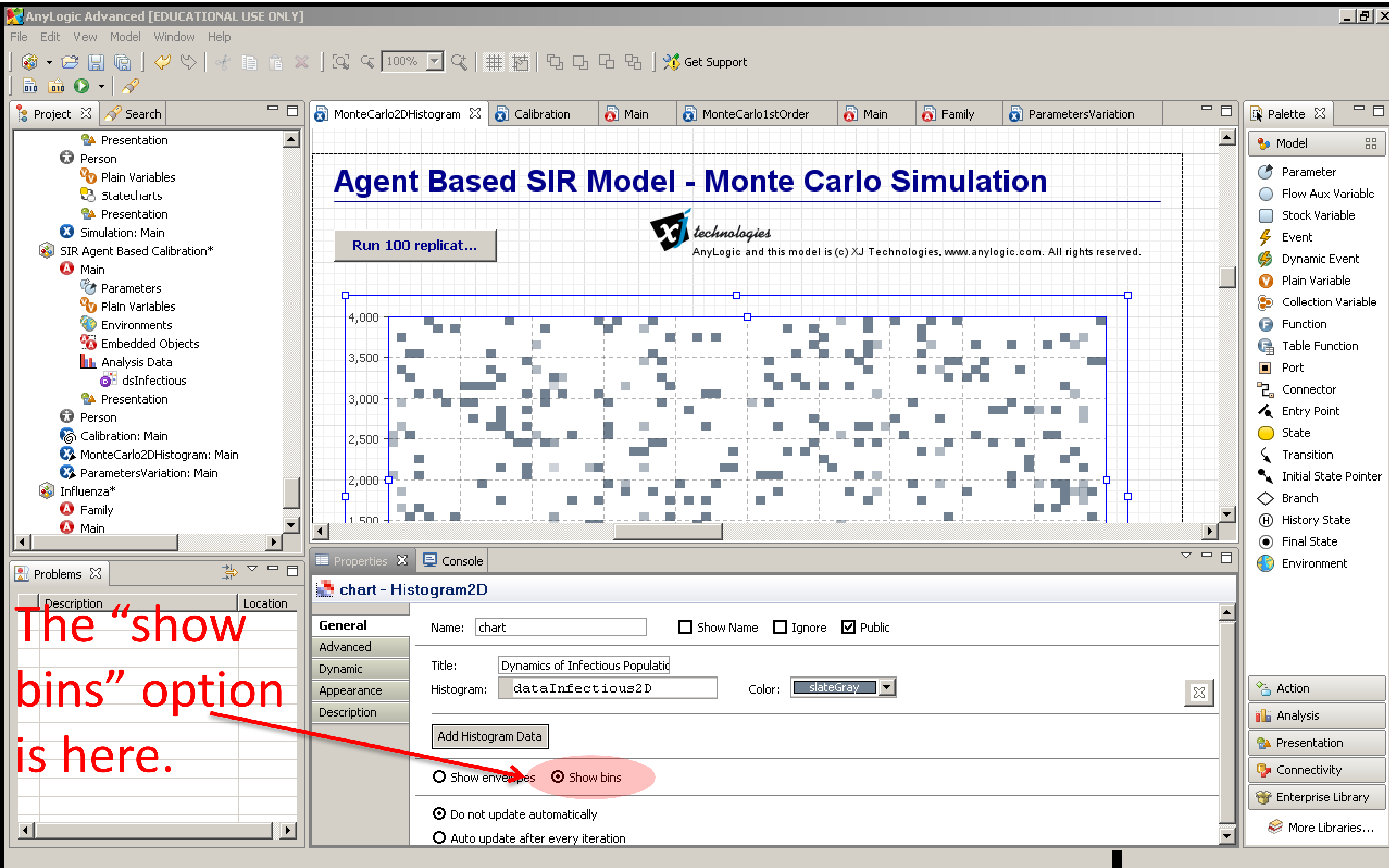
Example of “Show Envelopes” Output (Different Model)

A slice at this point in time would yield a something like a ***boxplot***. Note that the “whiskers” of the boxplot are *not* shown on the Histogram 2D chart (unless 0% and 100% fractiles are specified).

In contrast to a standard boxplot, the Histogram 2D chart can show arbitrarily many envelopes (rather than just quartiles, max, min and median). Note the contiguous nature of the envelopes.

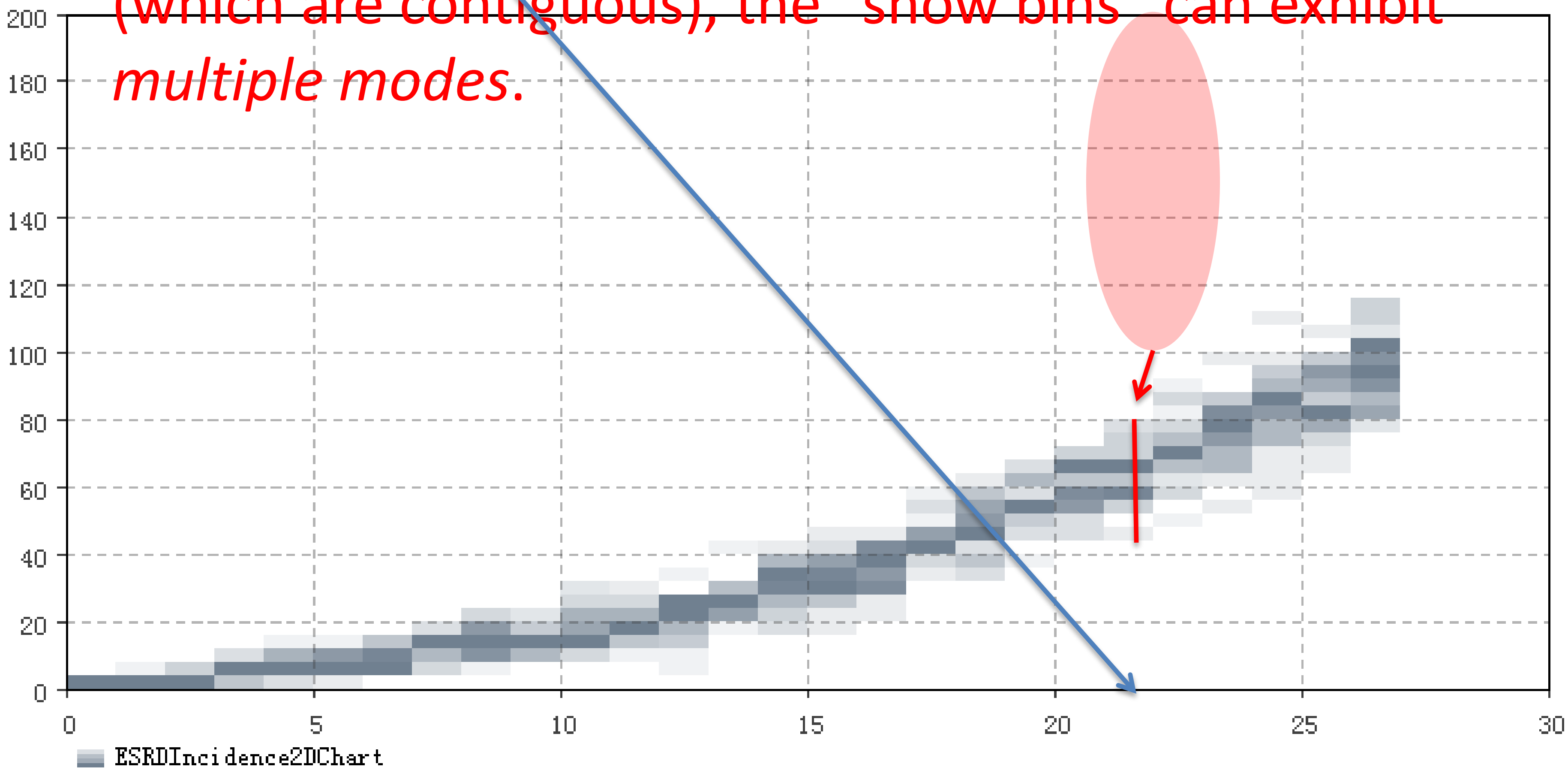


Show Bins Option

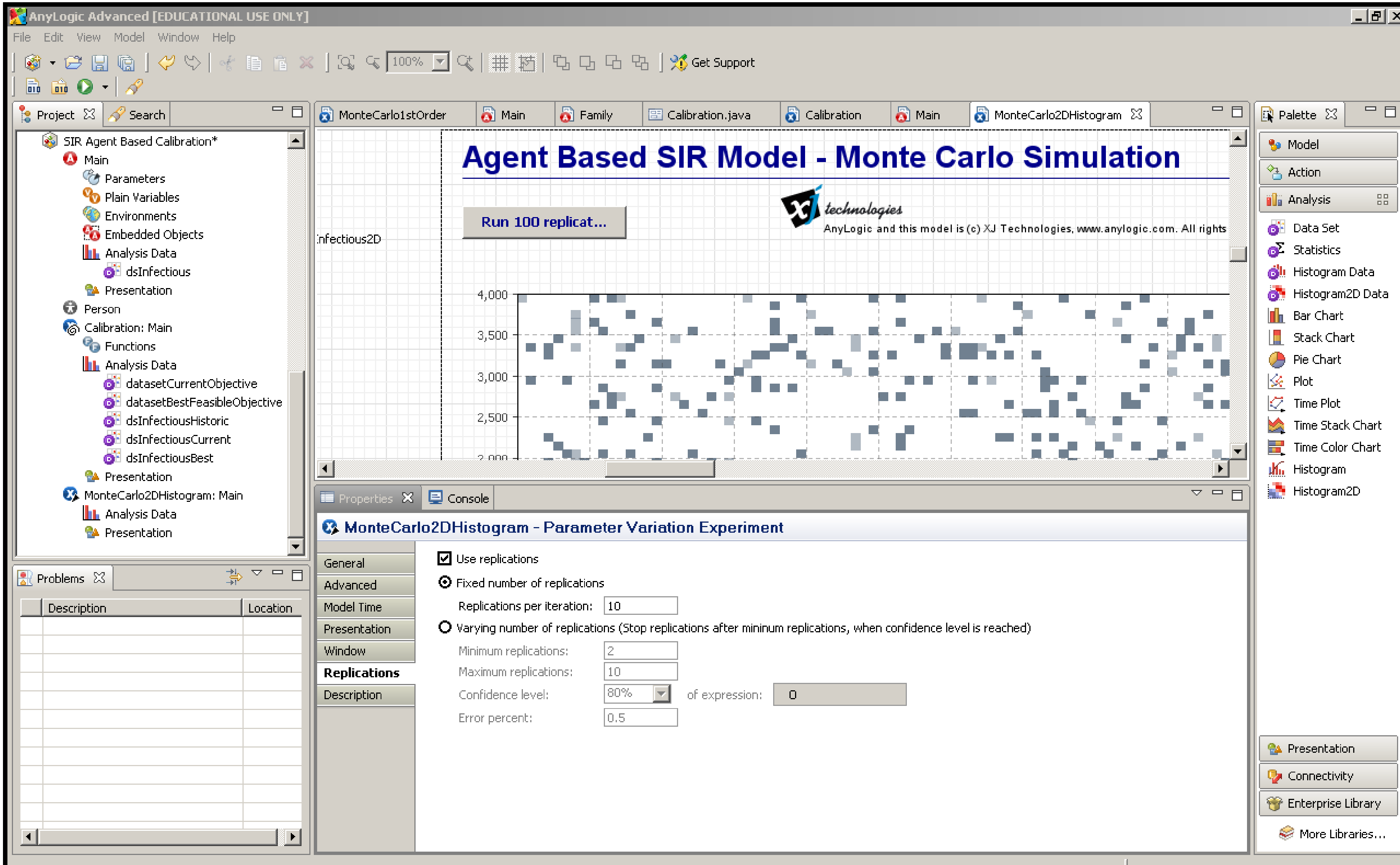


Example of “Show Bins” Output (Different Model)

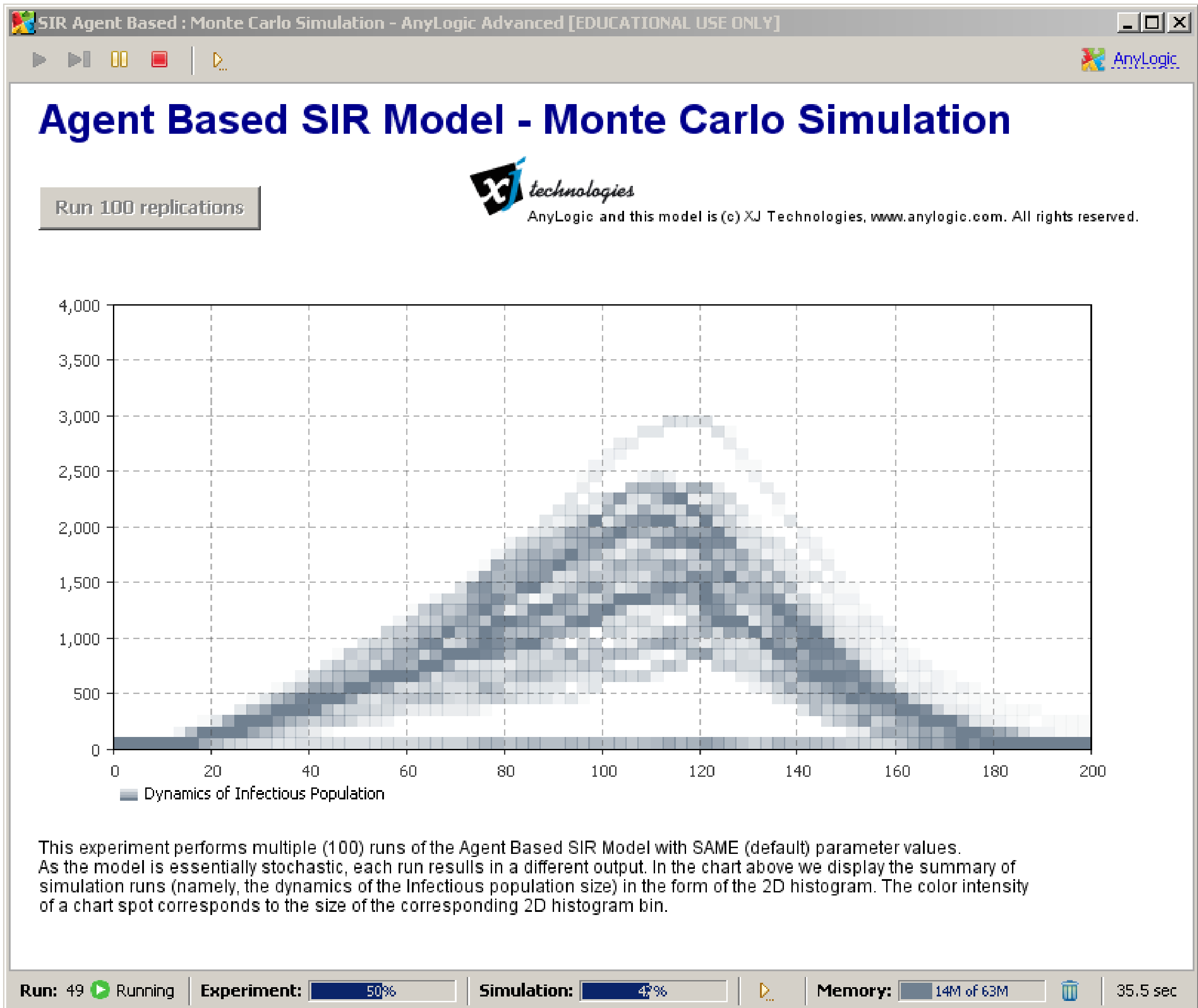
A slice at this point in time would yield a ***histogram***.
Note: In contrast to the situation for the envelopes (which are contiguous), the “show bins” can exhibit *multiple modes*.



Automatic Throttling of Monte Carlo Analyses



General Variety of Output



Reminder: Statistical Scaling

- Consider Taking the sample mean of *n* samples that vary independently around a mean
- If two samples x and y are independent samples of random variables X and Y, then Var[x+y]=Var[X]+Var[Y]
  - So if we have n indep. samples x_i from distribution X

$$Var\left(\sum_{i=1}^{n} x_i\right) = nVar\left(X\right)$$

- If we scale a random variable by a factor α, the standard deviation scales by the same factor of α => the variance scales by α^2
  - i.e. StdDev[αX]= α StdDev[X], Var[αX]= α^2 Var[X]

Statistics of Sample Mean

- Recall: Sample Mean: $$m = \frac{\sum_{i=1}^{n} x_i}{n}$$
- From the preceding, variance drops as 1/n

$$Var(m) = Var\left(\frac{\sum_{i=1}^{n} x_i}{n}\right) = \frac{Var\left(\sum_{i=1}^{n} x_i\right)}{n^2} = \frac{nVar(X)}{n^2} = \frac{Var(X)}{n}$$

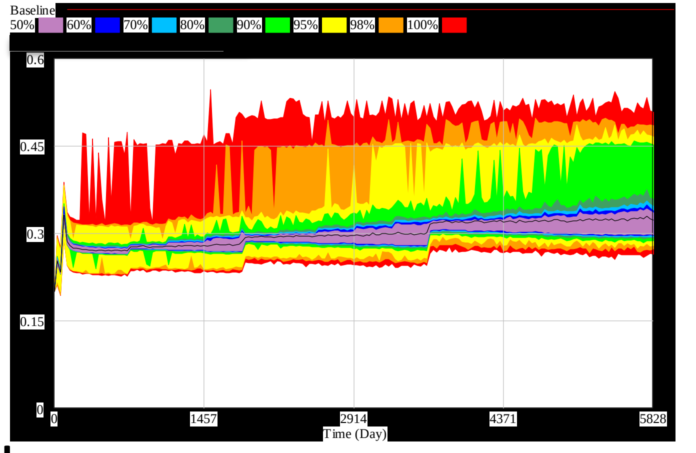
- This means that standard deviation for the sample mean of n samples drops as <u>1/sqrt(n)</u>

$$StdDev(m) = \sqrt{Var(m)} = \sqrt{\frac{Var(X)}{n}} = \sqrt{\frac{\left(StdDev(X)\right)^2}{n}} = \frac{StdDev(X)}{\sqrt{n}}$$

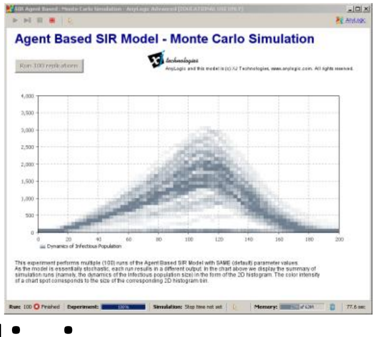
- So if we wish to divide the standard deviation of the sample mean by a factor of 2, we need to take 4x the number of Monte Carlo samples

Closing Question: How can we best adapt our policies to deal with ongoing uncertainty?

- We are dealing here with making decisions in an environment that changes over time
- This uncertainty could come from



  – Stochastic variability
  – Uncertainty regarding parameter values



- There is an incredibly vast # of possible policies
- Reminder: Can successfully integrate decision analysis & simulation to neatly handle such cases