Introduction to Stocks & Flows

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Reminder: Model Specification Mechanisms

Stock & Flow Models: "Hedgehog Knowledge"

- Small modeling vocabulary
- Power lies in combination of a few elements (stocks & flows)
- Analysis conducted predominantly in terms of elements of model vocabulary (values of stocks & flows)
- Directly maps onto crisp mathematical description (Ordinary Differential Equations)

Agent-Based Modeling: "Fox Knowledge"

- Large modeling vocabulary
- Different subsets of vocabulary used for different models
- Power in flexibility & combination of elements
- Variety in analysis focus
- Mathematical underpinnings differ
- In most cases, lacks transparent mapping to mathematical formulation

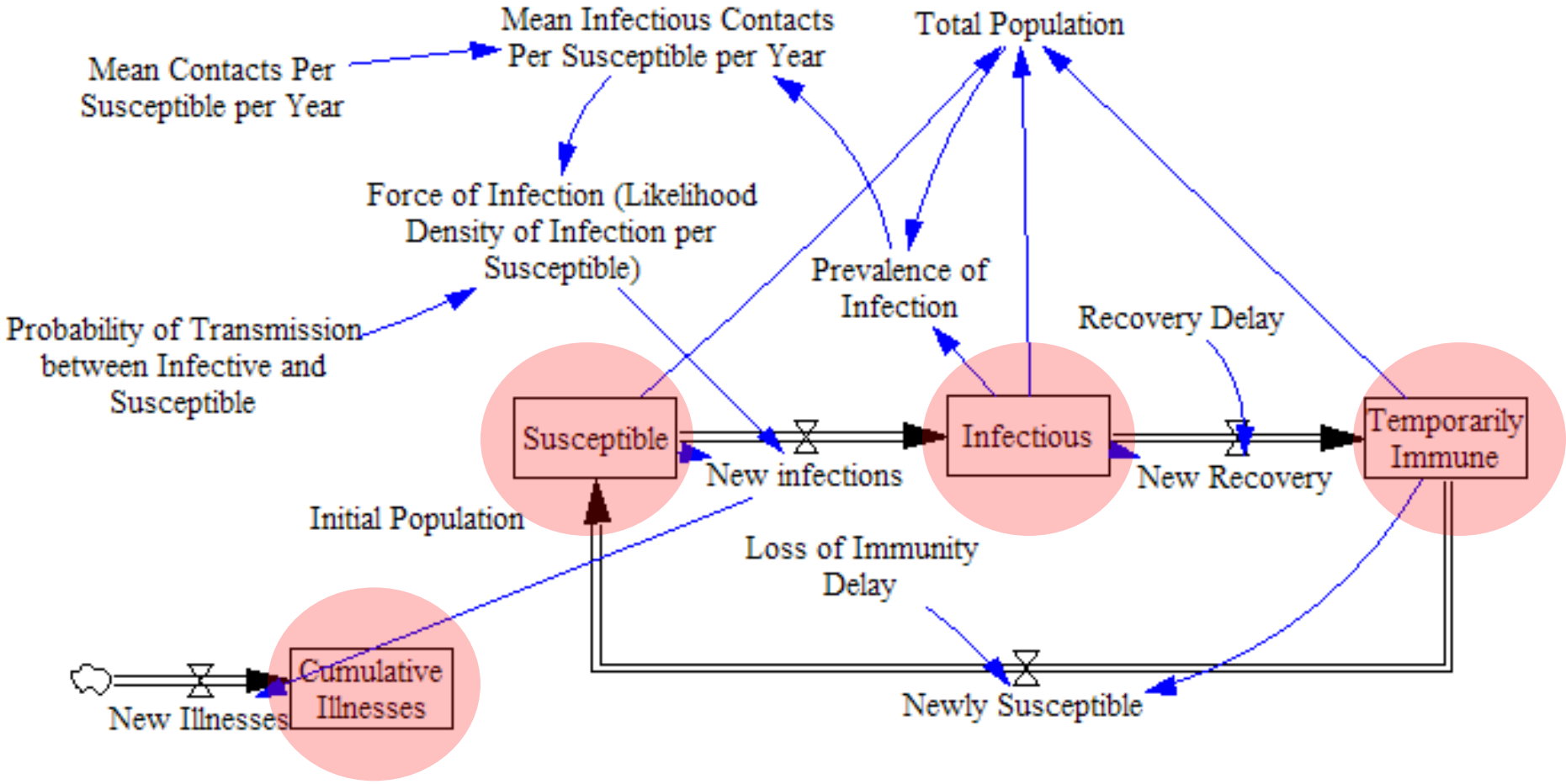
State of the System: Stocks ("Levels", "State Variables", "Compartments")

- Stocks (Levels) represent accumulations
  - These capture the "state of the system"
  - Mathematically, we will call these "state variables"
- These can be measured at *one instant in time*
- Stocks start with some initial value & are thereafter changed only by *flows* into & out of them
  - There are no inputs that immediately change stocks
- Stocks are the source of delay in a system
- In a stock & flow diagram, shown as ***rectangles***

Examples of Stocks

- Water in a tub or reservoir
- Planned work to be done
- Active (diagnosed, sanitized) software defects
- Yet unrecognized SW defects
- People of different types
  - Count of developers
  - {Susceptible, infective, immune} people
  - Women between the age of x and *y*
  - High-risk individuals
- Doses of medicine
- Money in bank account
- CO_2 in atmosphere
- Blood sugar
- Stored Energy
- Degree of belief in X
- Stockpiled vaccines
- Goods in a warehouse
- Beds in an emergency room
- Owned vehicles
- Healthcare workers

Example Model: Stocks



The Critical Role of Stocks in Dynamics

- Stocks determine current state of system
  - Stocks often provide the basis for making choices
- Stocks central to most disequilibria phenomena (buildup [accumulation], decay)
- Lead to inertia
- Give rise to delays

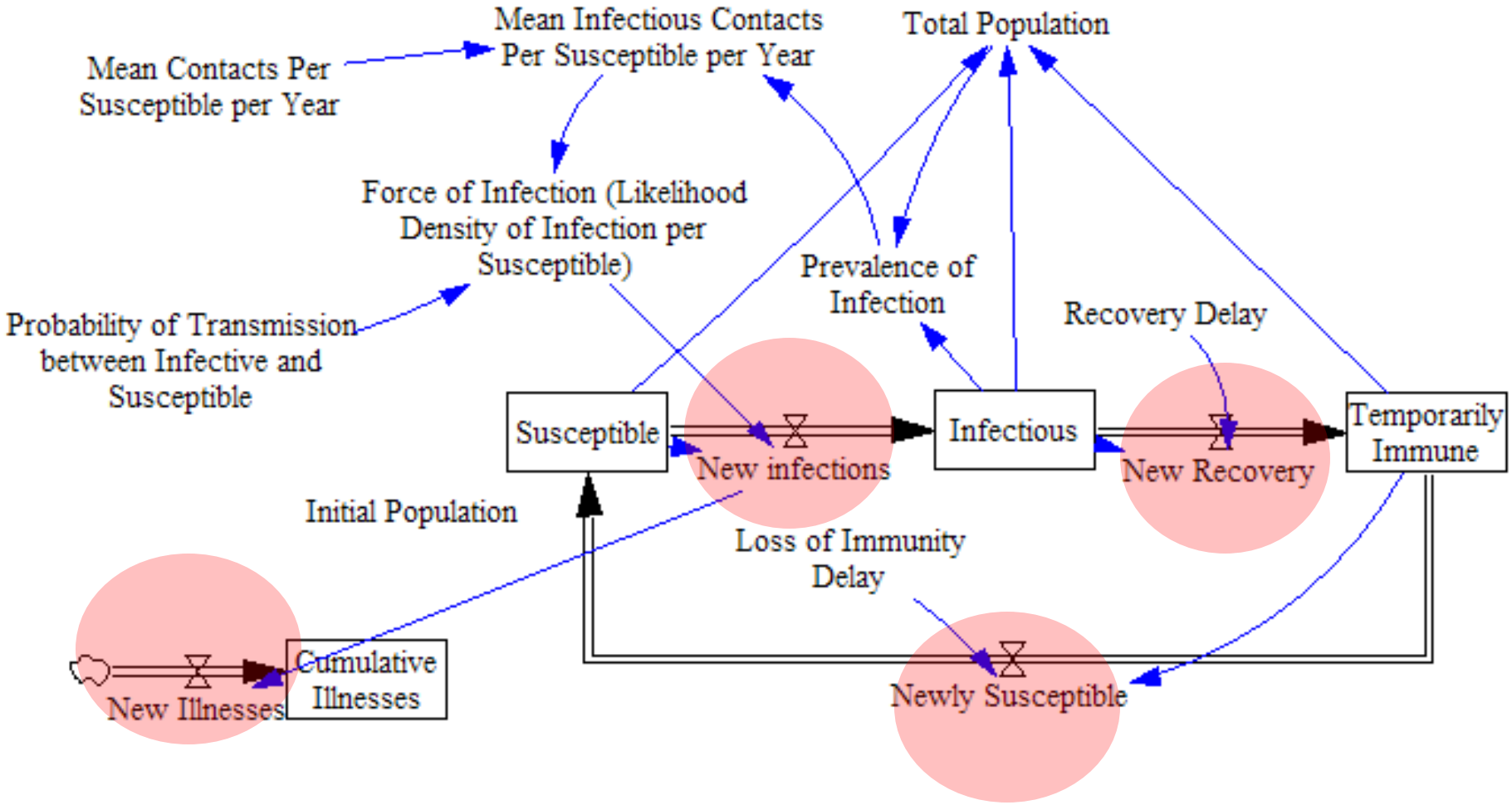
State Changes: Flows ("Fluxes", "Rates", "Derivatives")

- All changes to stocks occur via *flows*
- Always expressed per some unit time: If these flow into/out of a stock that keeps track of things of type *X (*e.g. persons), the rates are measured in *X*/(Time Unit) (e.g. persons/year, $/month, gallons/second)
- Typically measure over certain period of time (by considering accumulated quantity over a period of time)
  - e.g. Bug fix rate as bugs fixed/week, Incidence Rates is calculated by accumulating people over a year, project velocity in jellybeans/day, burn rate & revenue is $/Time, water flow is litres/minute
  - Can give estimate for rate at a point in time, but need period of time to measure

Examples of Flows

- Inflow or outflow of a bathtub (litres/minute)
- Rate of bug fixes (bugs/week)
- Rate of code production (e.g. Lines of code/month)
- Rate of incident cases (e.g. people/month)
- Rate of mortality (e.g. people/year)
- Rate of births (e.g. babies/year)
- Rate of treatment (people/day)
- Rate of caloric consumption (kcal/day)
- Rate of pregnancies (pregnancies/month)
- Reactivation Rate (# of TB cases reactivating per unit time)
- Revenue ($/month)
- Spending rate ($/month)
- Power (Watts)
- Rate of energy expenditure
- Vehicle sales
- Vaccine sales
- Shipping rate of goods

Example Model: Flows



Flows 2

- We can ask conceptually *about* the rate *at* any given point in time – and may change over time
  - Measuring it would have to be over some period
- When speaking about "rates" for flows, we always mean a *rate of change over time (*something measured as X/*Unit Time*)
  - Not all things called "rates" are flows
    - Exchange rate
    - Prevalence rate
    - Rate of return

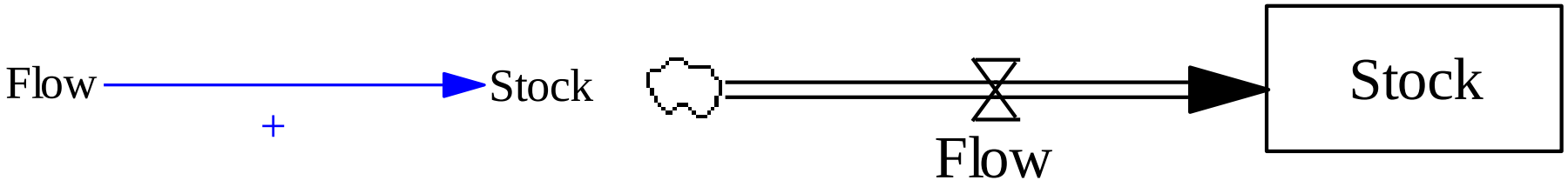
Distinguishing Stocks & Flows: Heuristics

- To determine if a quantity is a stock or flow:
  - "Snapshot" test: If you were only to consider a moment in time (a "snapshot" of the system), could the quantity be clearly quantified by the information available at that moment?
    - If yes, stock (cannot quantify a value of a flow using only the information for an instant – must measure over time)
  - "Time unit change" test: If we were to change the unit by which we measure time, would the numeric value of the quantity change?
    - If yes, quite likely to be a *flow* (exception: beliefs about flows)
  - "Accumulation" test: Is this quantity an accumulation of the time-varying values of other quantities?
    - If yes, stock

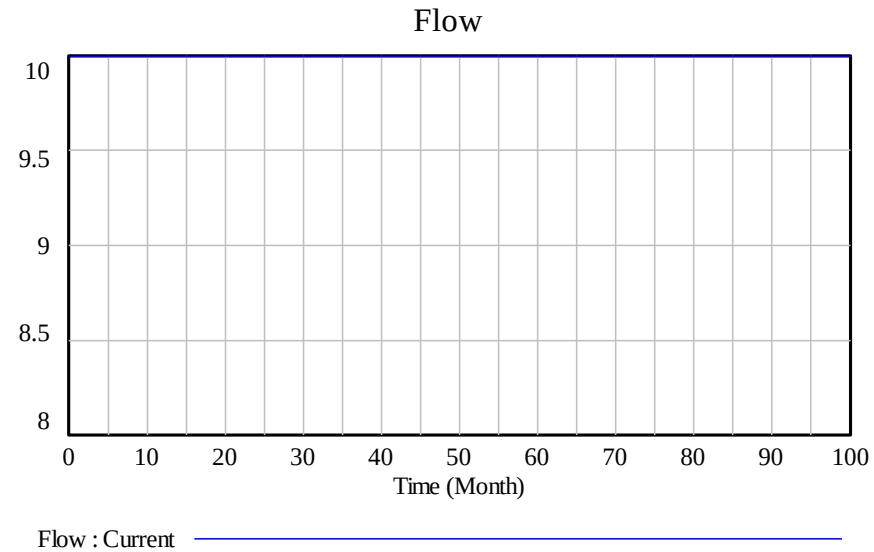
Exercise: Stocks or Flows?

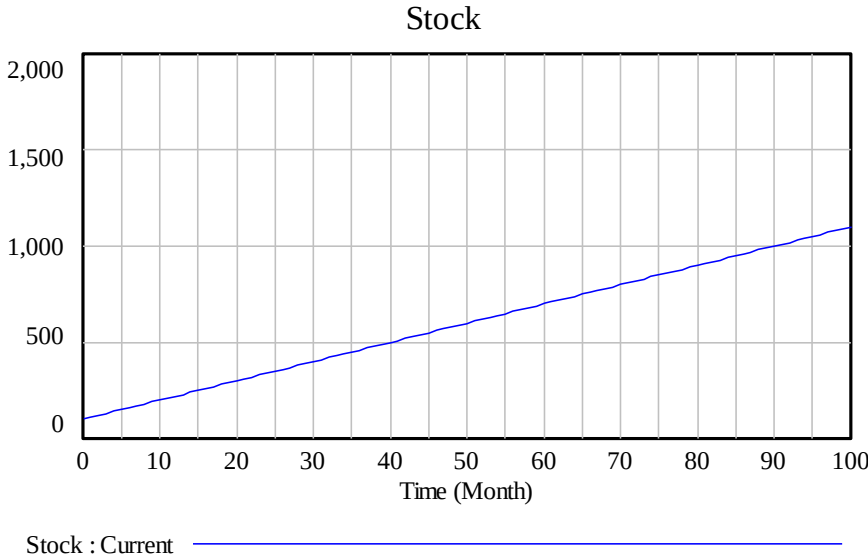
- Account balance
- Income
- Incidence
- Prevalence
- Temperature
- Births
- Profits
- Interest
- Principal
- Shipments
- Car accidents
- Patients on dialysis
- Deaths
- Heart attacks
- Arrests
- Police
- Patients in hospital
- Hospital admissions
- Position
- Speed

Key Component: Stock & Flow

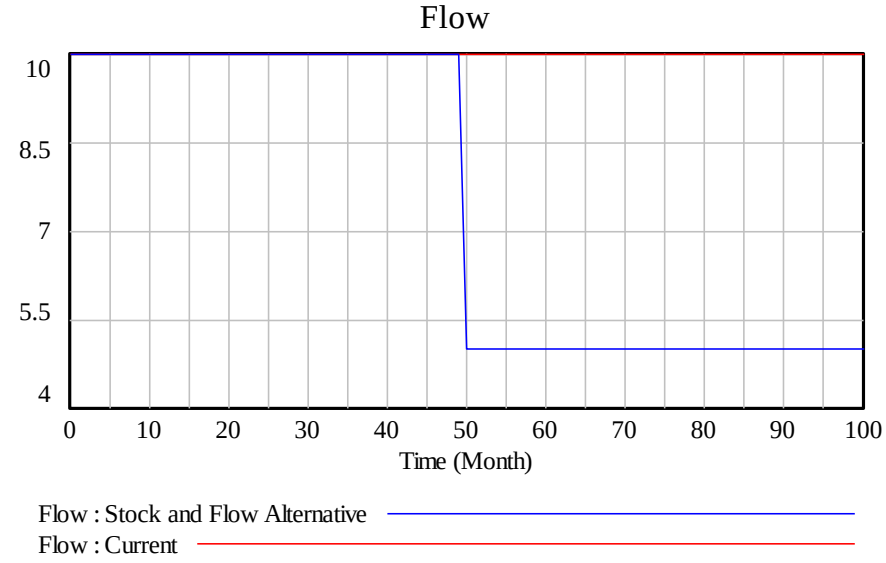


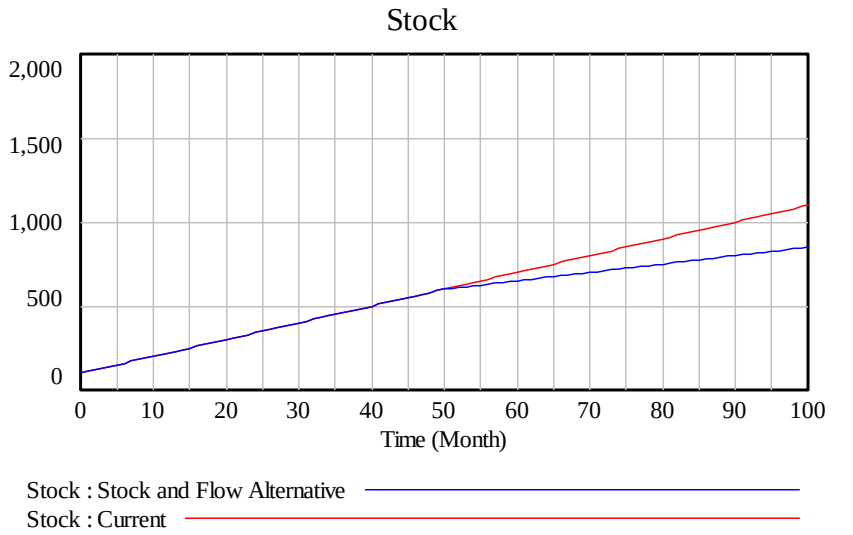
Net Flow Impact on Stock





Impact of Lowering Flow (Rate) to 5/Month?





Loops & Stocks

- Causation does not effect big change instantaneously
  - Loops are not instantaneous
- Stocks only change by changes to the flows into & out of them
  - There are no inputs that immediately change stocks
- All causal loops must involve at least one stock
  - The state of the world must change as part of the process
  - Absent a stock, loop would be instantaneous

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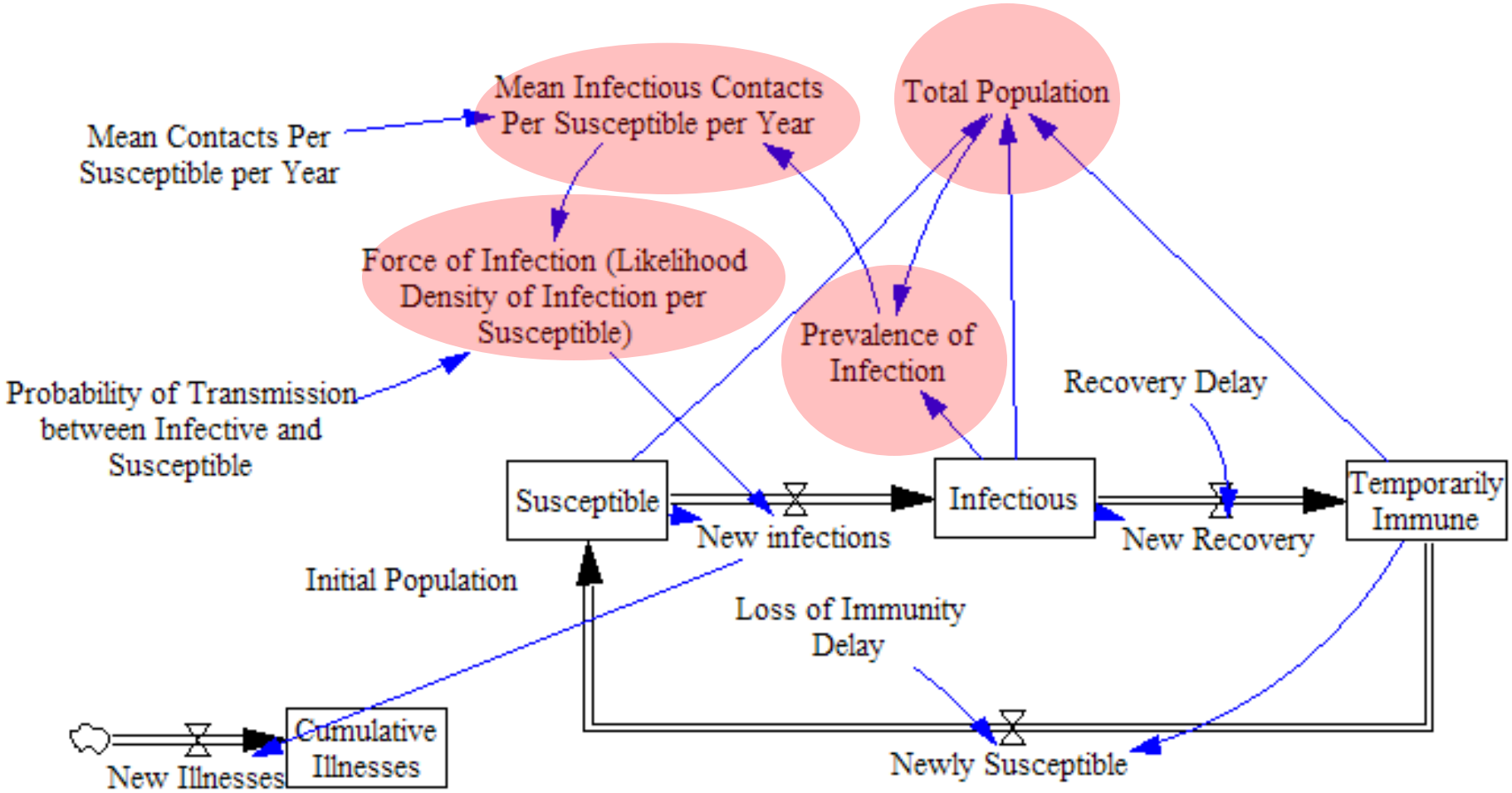
Auxiliary Variables

- Auxiliary variables are convenience names we give to concepts that can be defined in terms of expressions involving stocks/flows at current time
  - Adding or eliminating an auxiliary variable does not change the mathematical structure of the system
- Critical for model transparency
  - Can be reused at many places
  - References to auxiliary variables prevents need for modeler to think about all of details of definition
- Enhanced modifiability: Single place to define
- Convenient for reporting (graphing, tables) & analyzing model dynamics

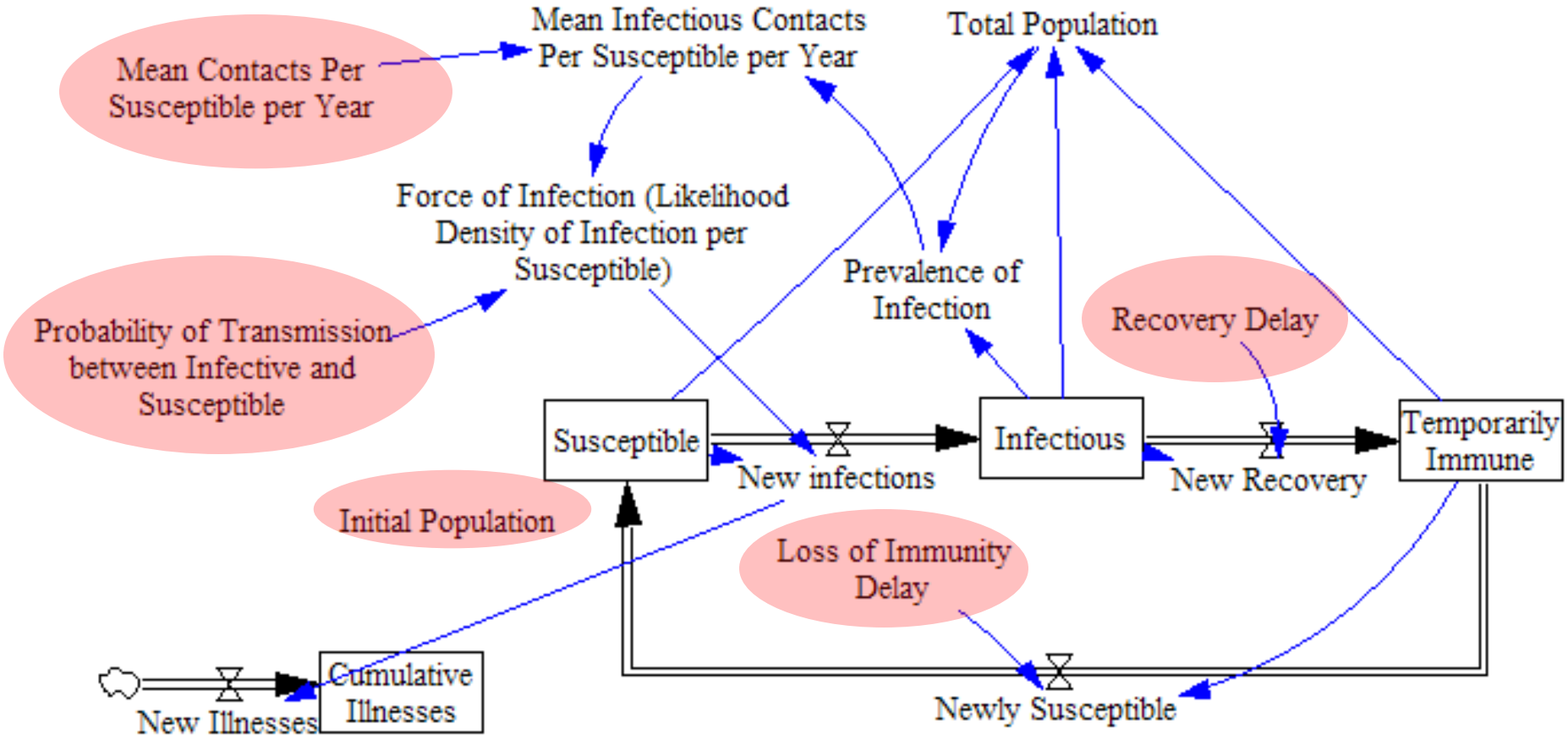
Example Model: Auxiliary Variables



Constants & Time Series Parameters

- For similar reasons to auxiliary variables, we give names to
  - Model constants
  - Time series

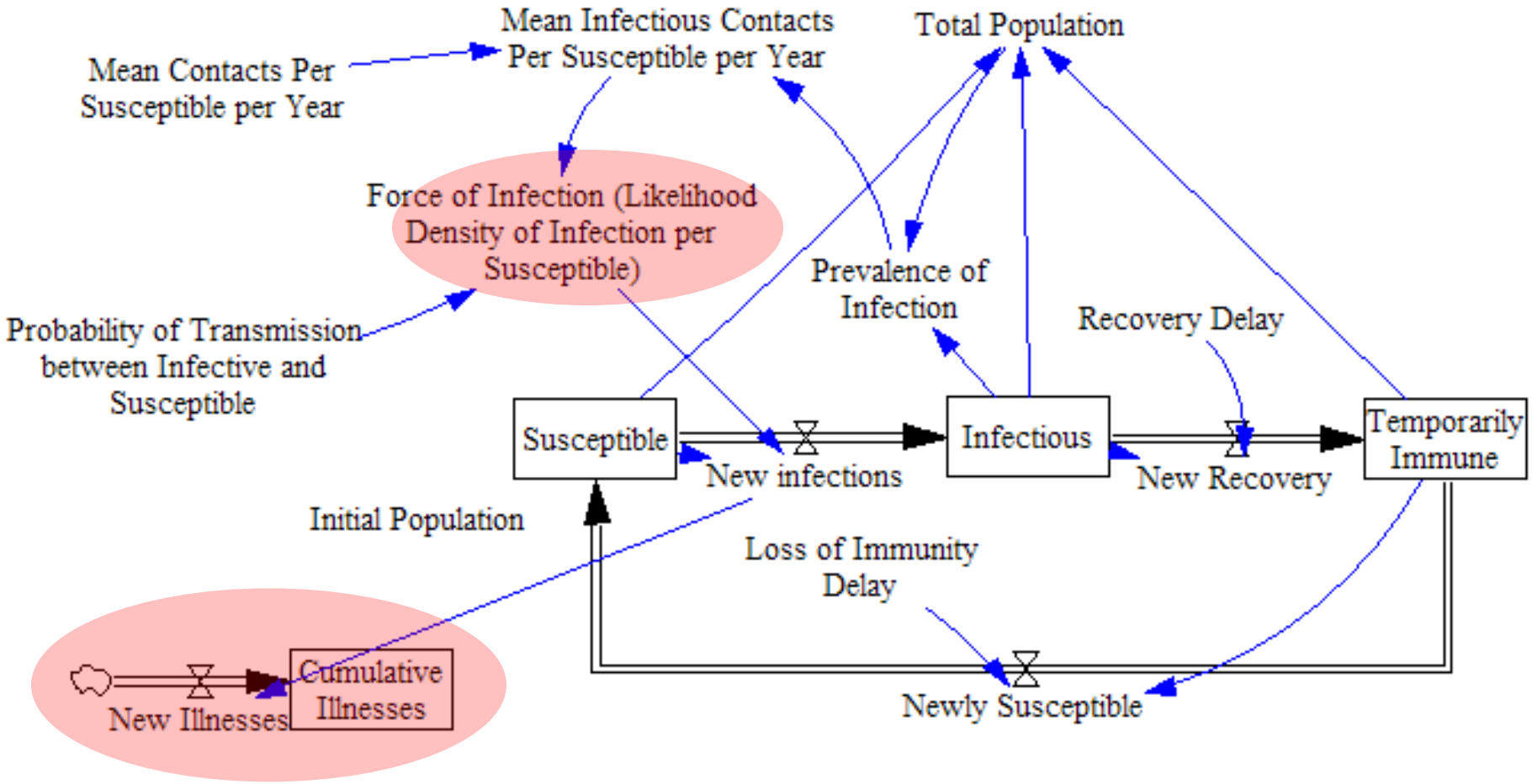
Example Model: Parameters



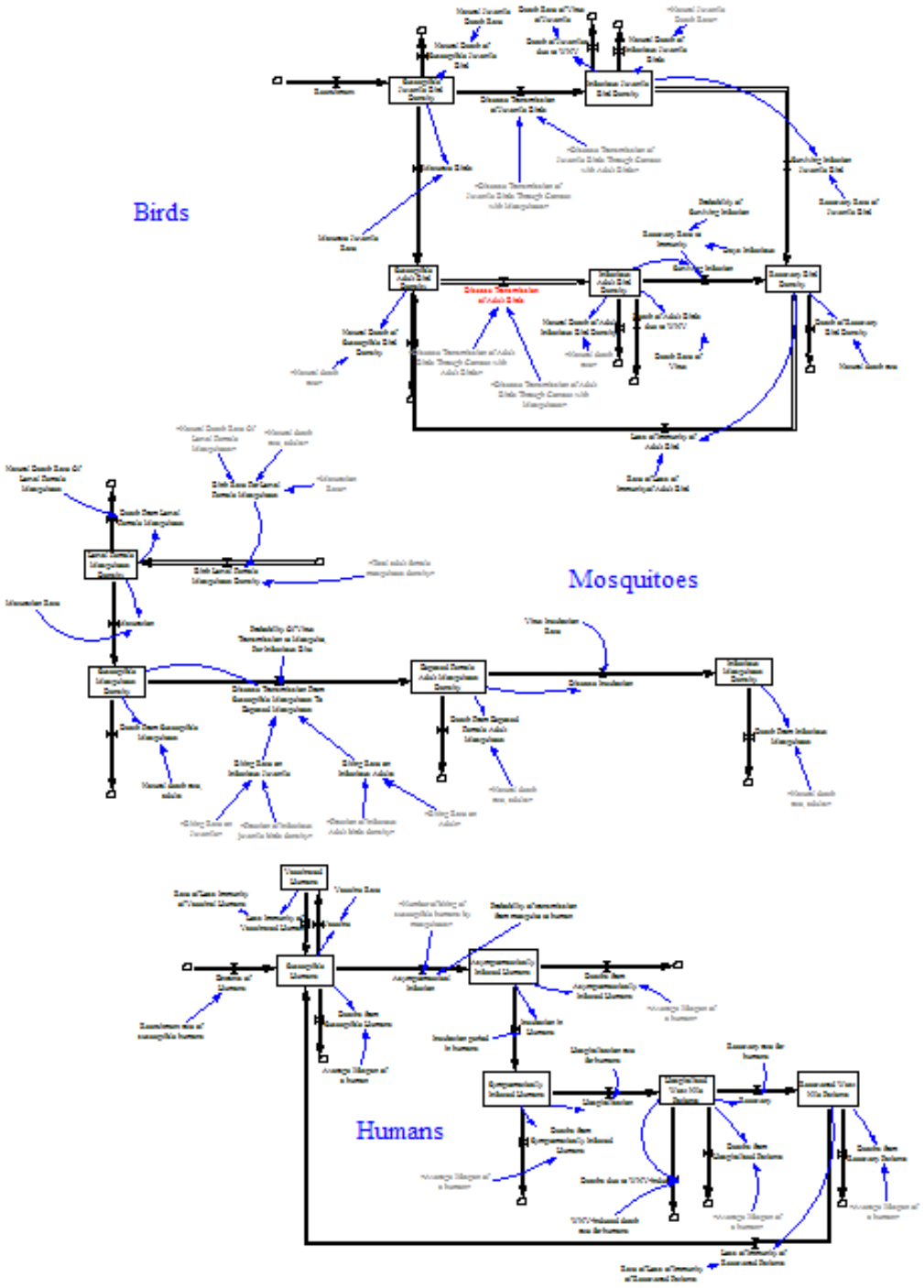
Stocks & Flows Compared with Markov Models

- Open population
  - Births
  - Deaths
- Non-constant likelihood (density) of transitions
  - Likelihood of leaving a stock per unit time can depend on other stocks
    - Force of Infection (likelihood of susceptible becoming infected) can depend on prevalence of illness
    - Likelihood of initiating smoking could depend on accumulated current or former smokers
- Multiple types of stocks
  - e.g. costs, QALYs, hosts & reservoir species, etc.
- Continuous time

Distinctive Stock & Flow Features



Multi-Species Model (West Nile Virus)

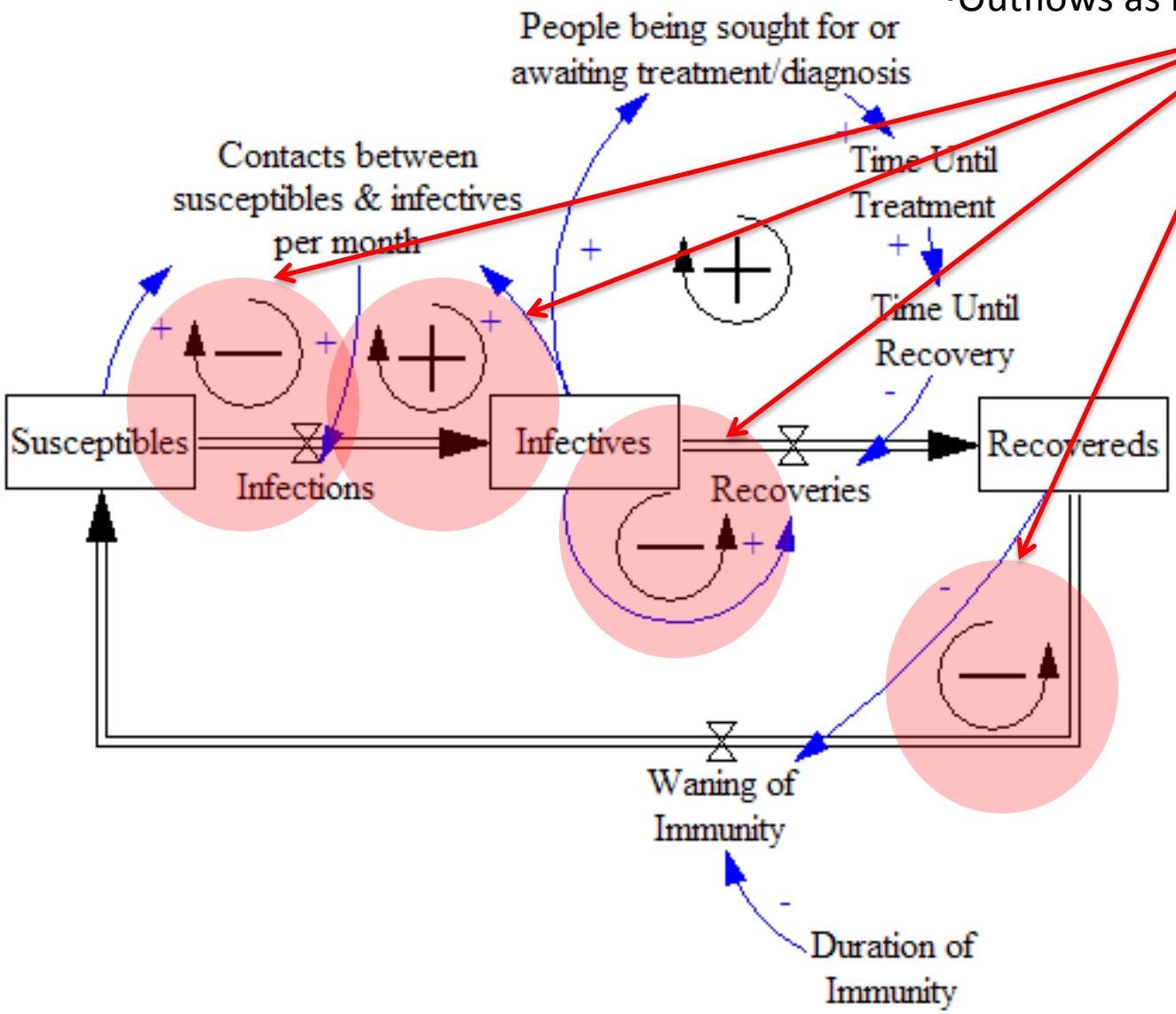


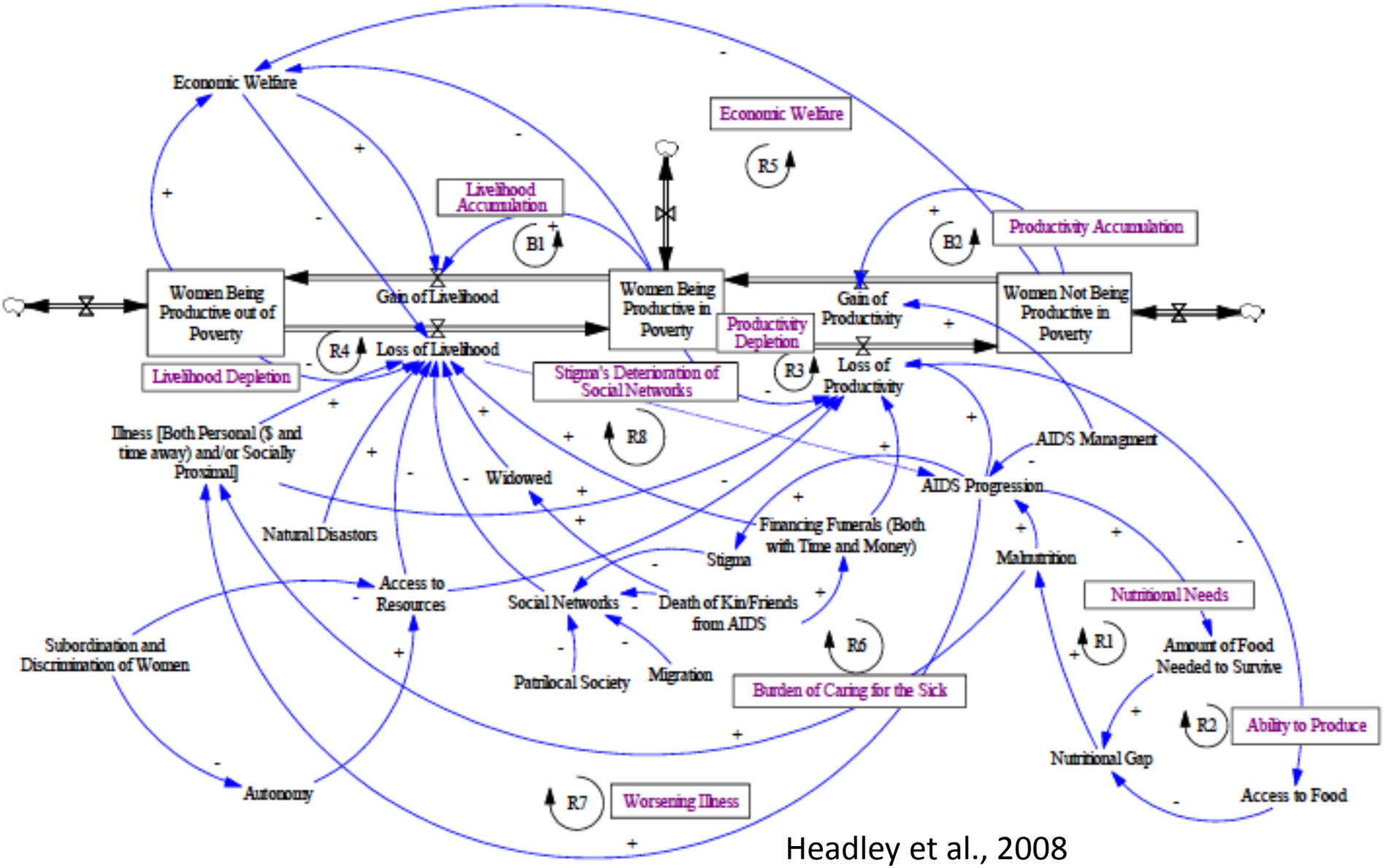
Refinement of Causal Loop Diagrams: System Structure Diagrams

- Still essentially a qualitative model, but less ambiguous
  - By clearly distinguish stocks & flows, this helps reduce the artifactual loops discussed with CLDs
- Combine causal loops diagram elements with stock & flow structure
- If complete, all loops will go “through a stock”
  - Loop goes into the flow of a stock (as one variable in the diagram)
  - Loop comes comes out of stock (as next variable in diagram)

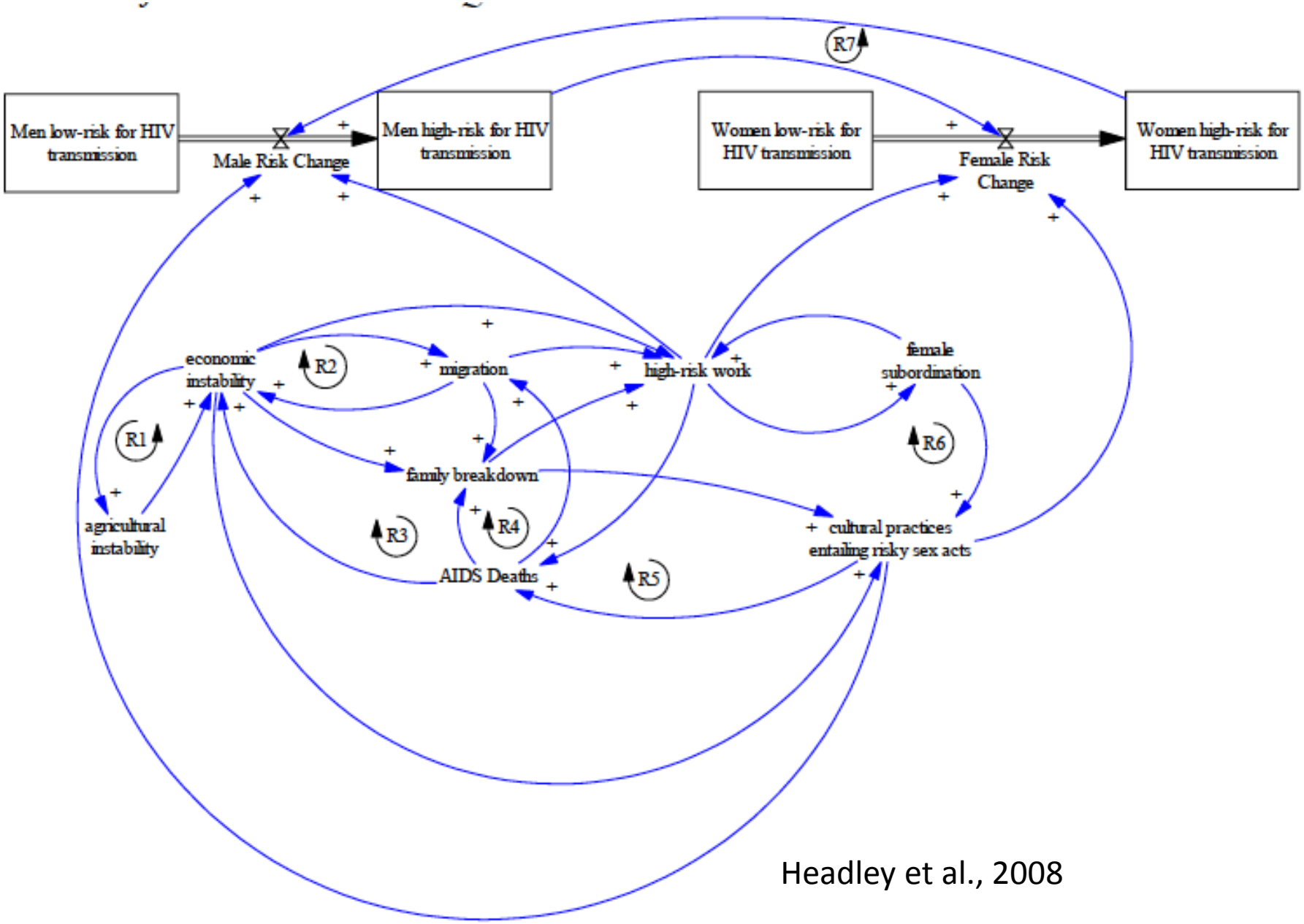
Example System Structure Diagram

Note treatment of flows as links from flow to stock
- Inflows as positive links
- Outflows as negative links





Headley et al., 2008



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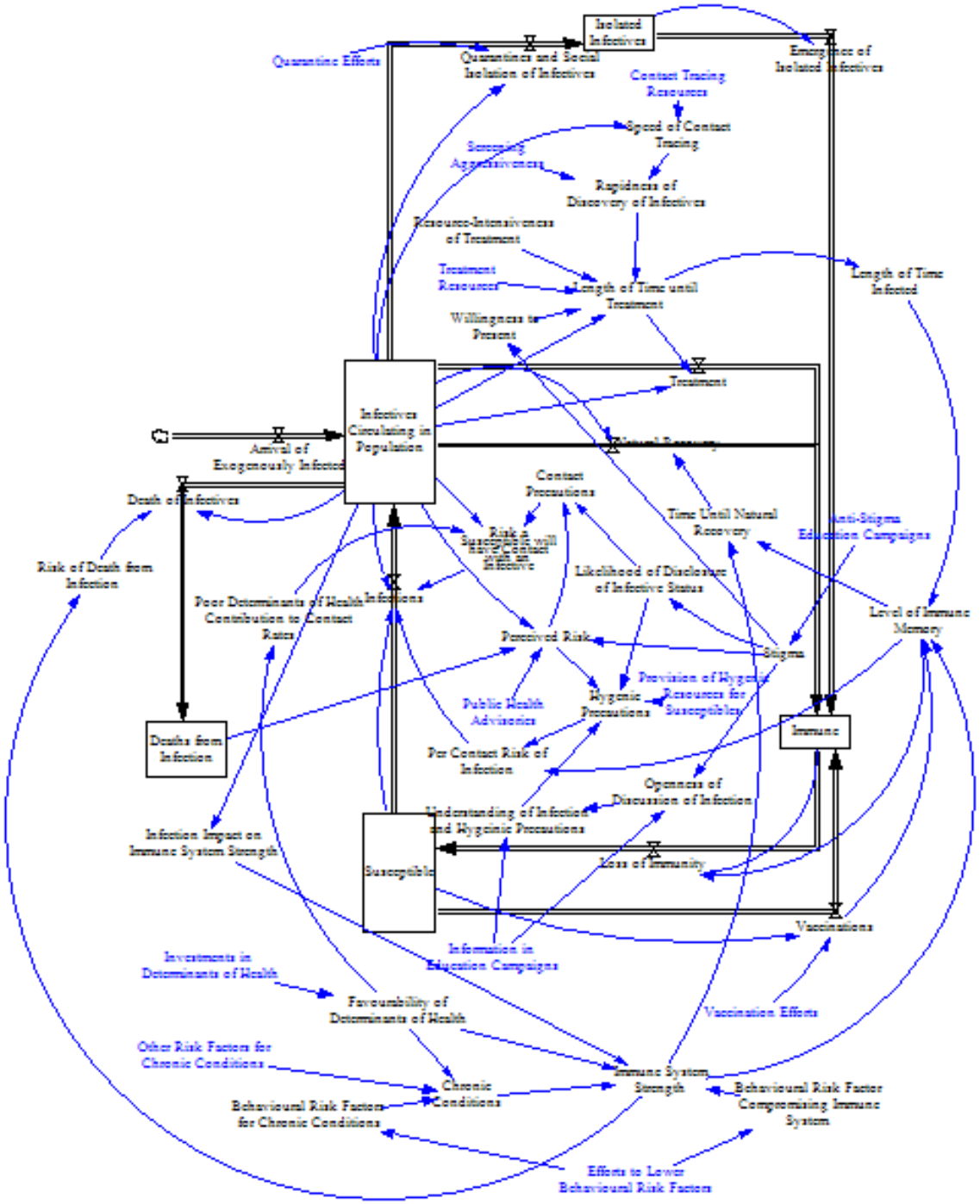
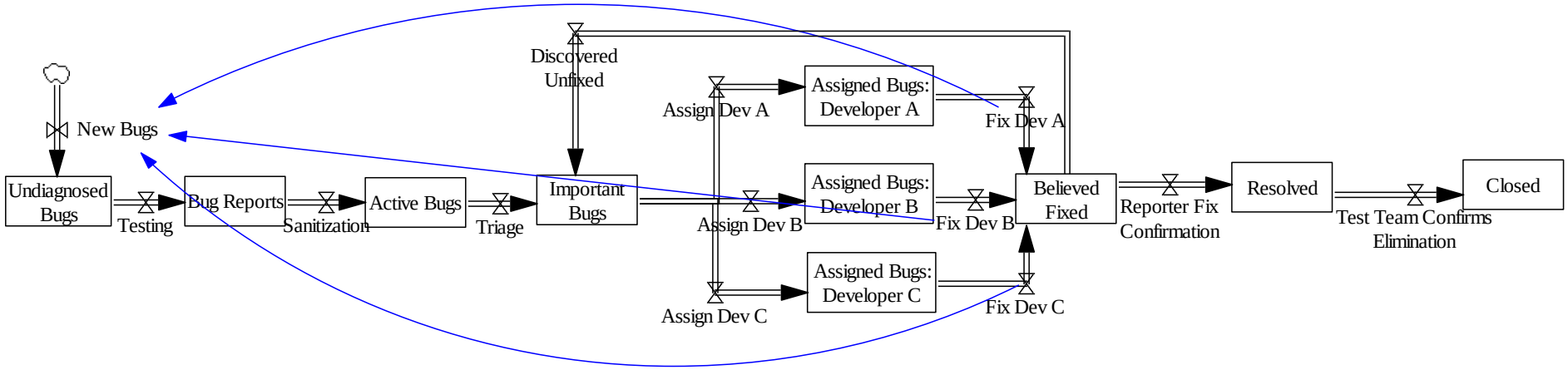

Isolated Infectives
Quarantine Efforts
Quarantines and Social Isolation of Infectives
Emergence of Isolated Infectives
Contact Tracing Resources
Speed of Contact Tracing
Screening Appropriateness
Rapidness of Discovery of Infectives
Resource-Intensiveness of Treatment
Treatment Resources
Length of Time until Treatment
Length of Time Infected
Willingness to Present
Treatment
Infectives Circulating in Population
Natural Recovery
Arrival of Exogenously Infected
Death of Infectives
Contact Precautions
Time Until Natural Recovery
Anti-Stigma Education Campaigns
Risk a Susceptible will have Contact with an Infective
Risk of Death from Infection
Infections
Likelihood of Disclosure of Infective Status
Poor Determinants of Health Contribution to Contact Rates
Level of Immune Memory
Perceived Risk
Stigma
Provision of Hygienic Resources for Susceptibles
Public Health Advisories
Hygienic Precautions
Deaths from Infection
Immune
Per Contact Risk of Infection
Openness of Discussion of Infection
Understanding of Infection and Hygienic Precautions
Infection Impact on Immune System Strength
Susceptible
Loss of Immunity
Vaccinations
Information in Education Campaigns
Investments in Determinants of Health
Favourability of Determinants of Health
Vaccination Efforts
Other Risk Factors for Chronic Conditions
Immune System Strength
Chronic Conditions
Behavioural Risk Factor Compromising Immune System
Behavioural Risk Factors for Chronic Conditions
Efforts to Lower Behavioural Risk Factors

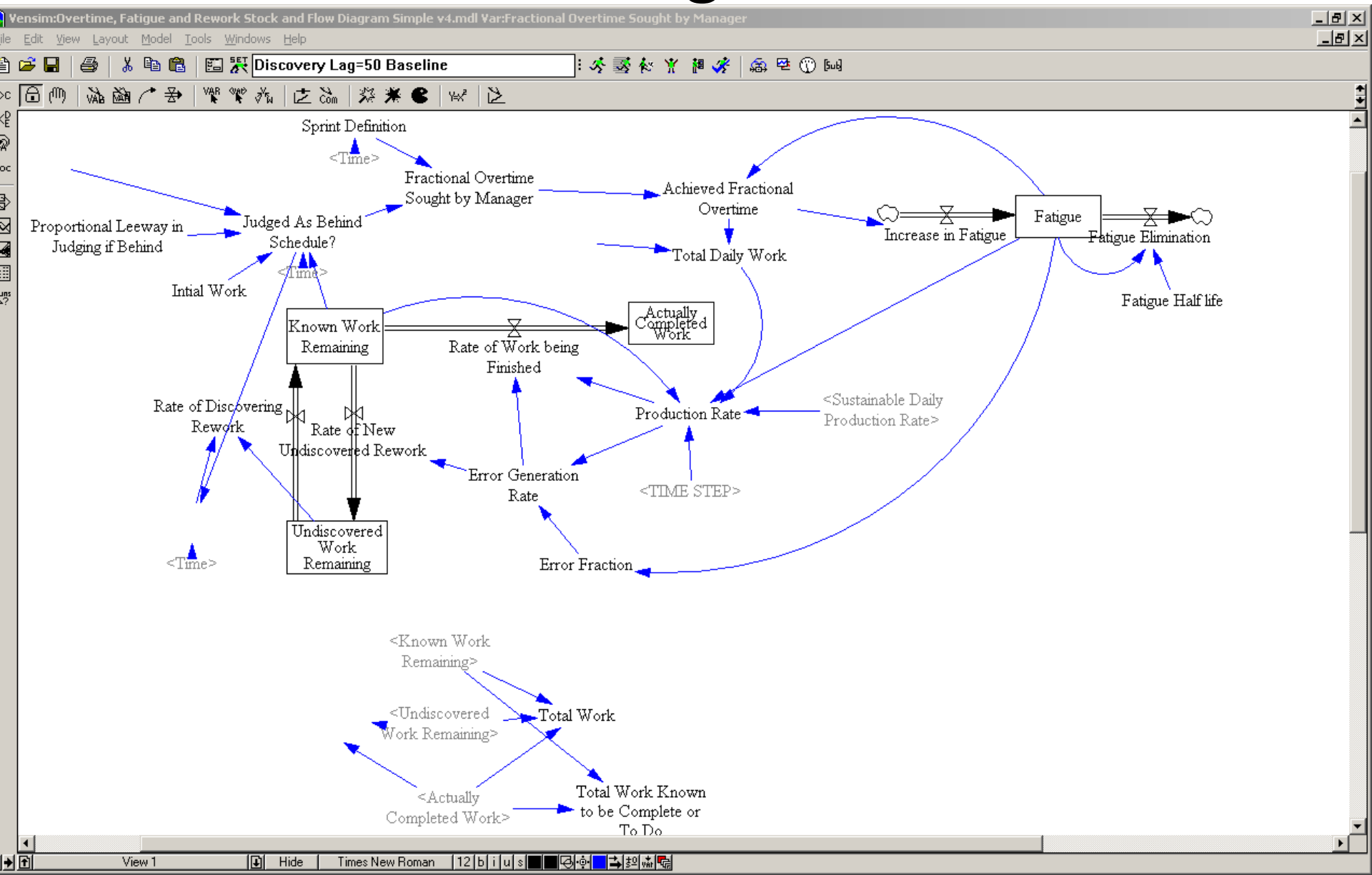
Reminder: Bug Resolution Pipeline



Some Stocks & Flows: Software Development

- Developers
  - Stocks: Rookie Developers , Seasoned Developers
  - Flows: Hiring, Leaves of Absence, Layoffs
- Productivity
  - Stocks
    - Work planned
    - Work accomplished
    - Work discarded
    - Fatigue
    - Active { requirements, design, coding } defects
    - Undiagnosed { requirements, design, coding } defects
  - Flows: Testing & peer review finding defects, work accomplished, fixing defects, etc.

Stock and Flow Dynamics of Work and Fatigue



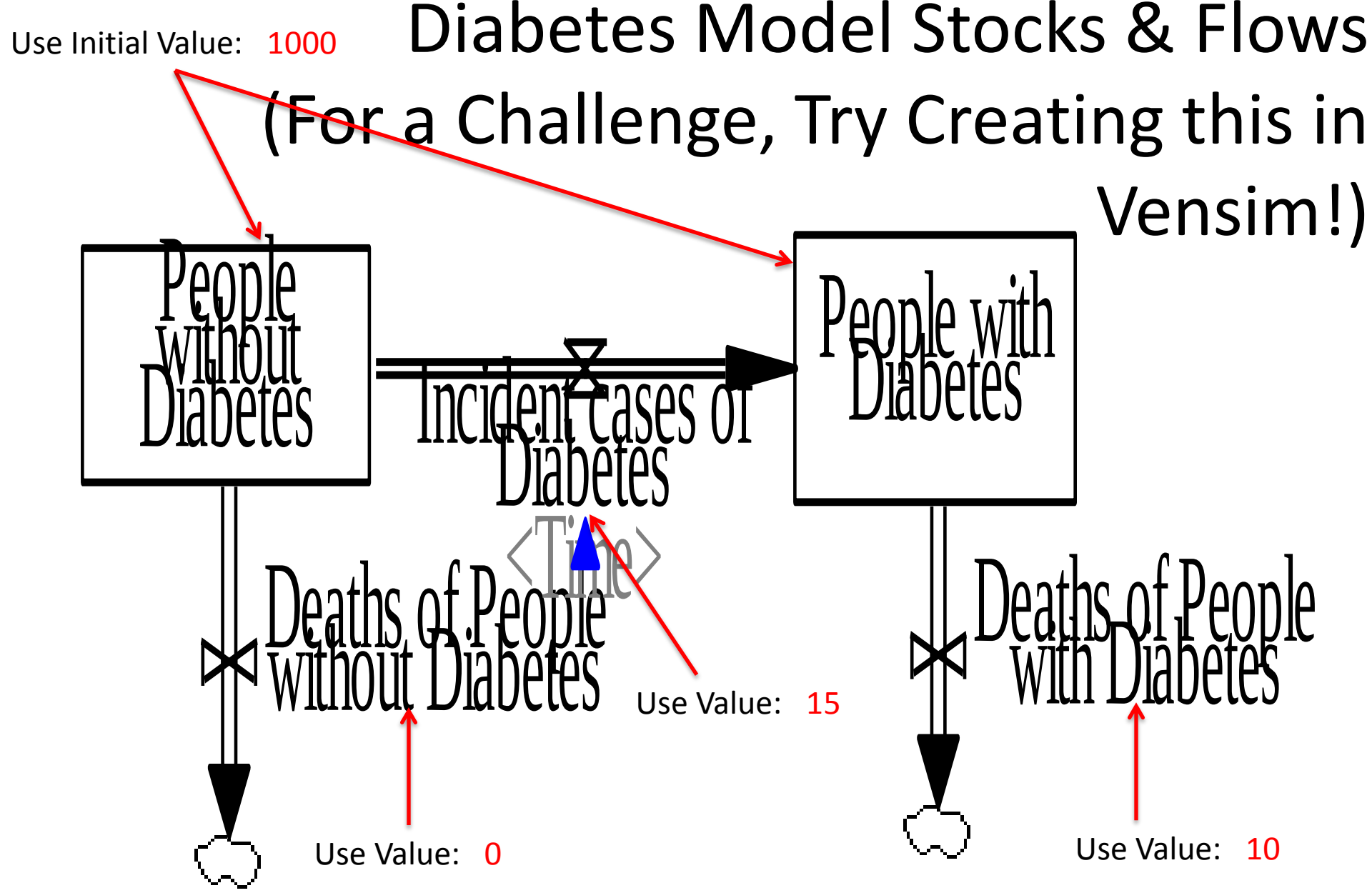
Stocks & Flows: Diabetes

- Assume diabetes is not curable
- Stocks:
  - People without diabetes (at different stages of risk?)
  - People with diabetes
- Flows
  - Incident cases (both diagnosed & undiagnosed!)
  - Deaths from both stocks

Stocks & Flows: Tuberculosis

- Assume that TB infection cannot be totally eliminated
- Stocks
  - Susceptible people
  - Immunized people
  - People with latent TB infection
  - People with active TB infection
- Flows
  - People becoming latently infected
  - People being vaccinated
  - People with infection going to Active TB (“primary progression”
  - People with infection going on to latent TB
  - People with secondary infection going on to active TB
  - Deaths from each stock

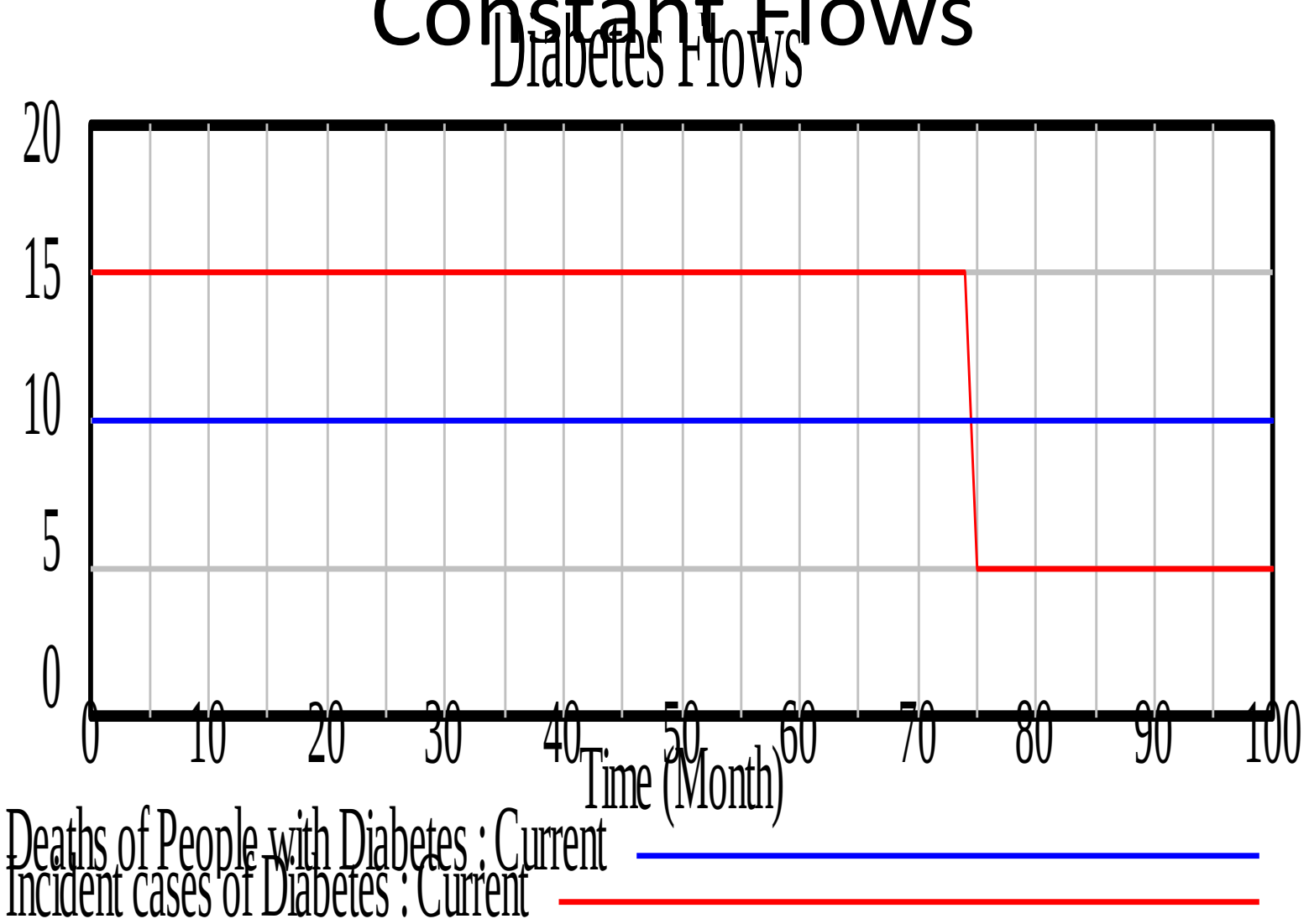
Diabetes Model Stocks & Flows (For a Challenge, Try Creating this in Vensim!)



Interactive Steps

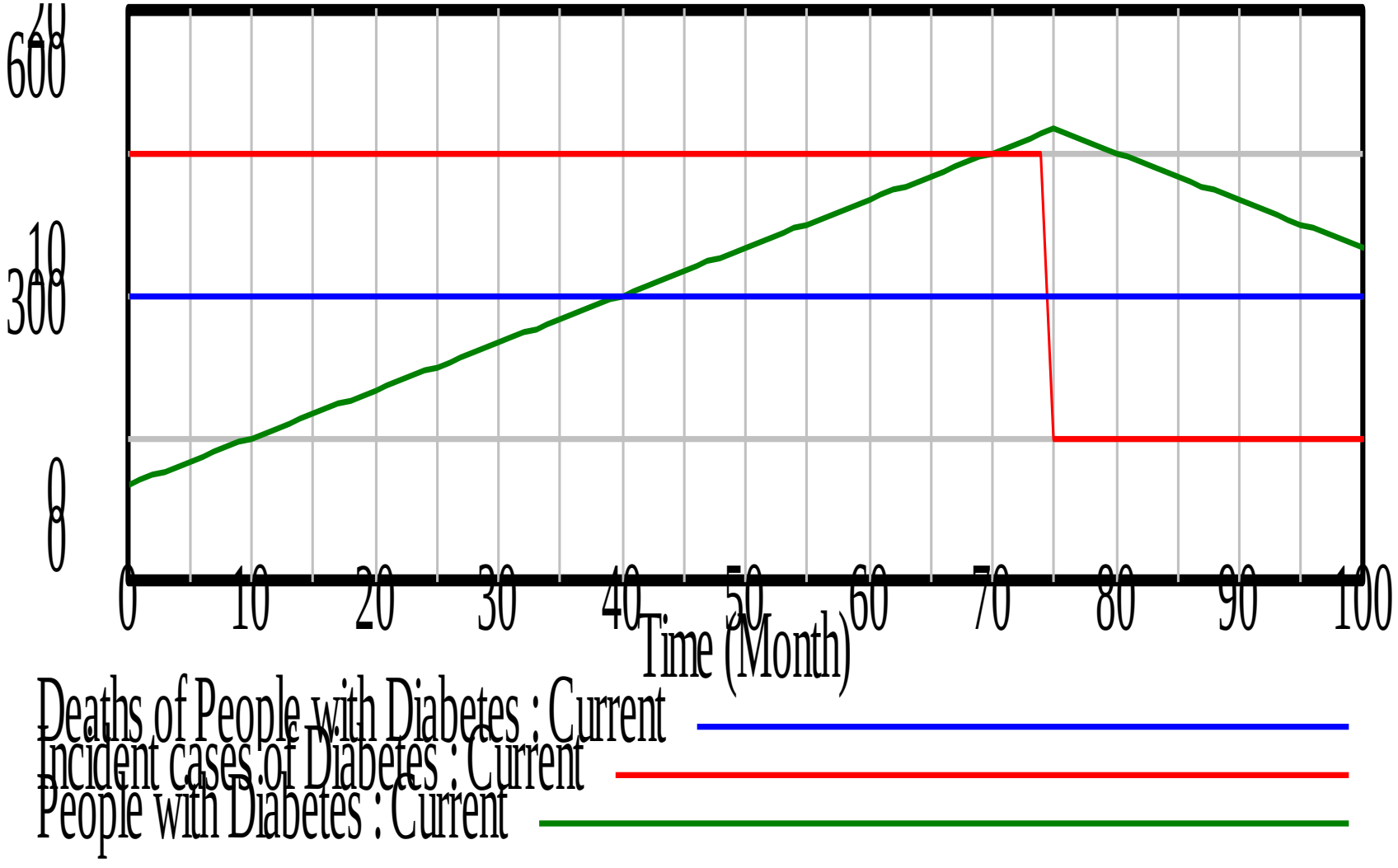
- View flows and stocks – does this make sense?
- Hitch up constant “auxiliary” variables to flows
- How does changing constant variables change the stock?

Constant Flows

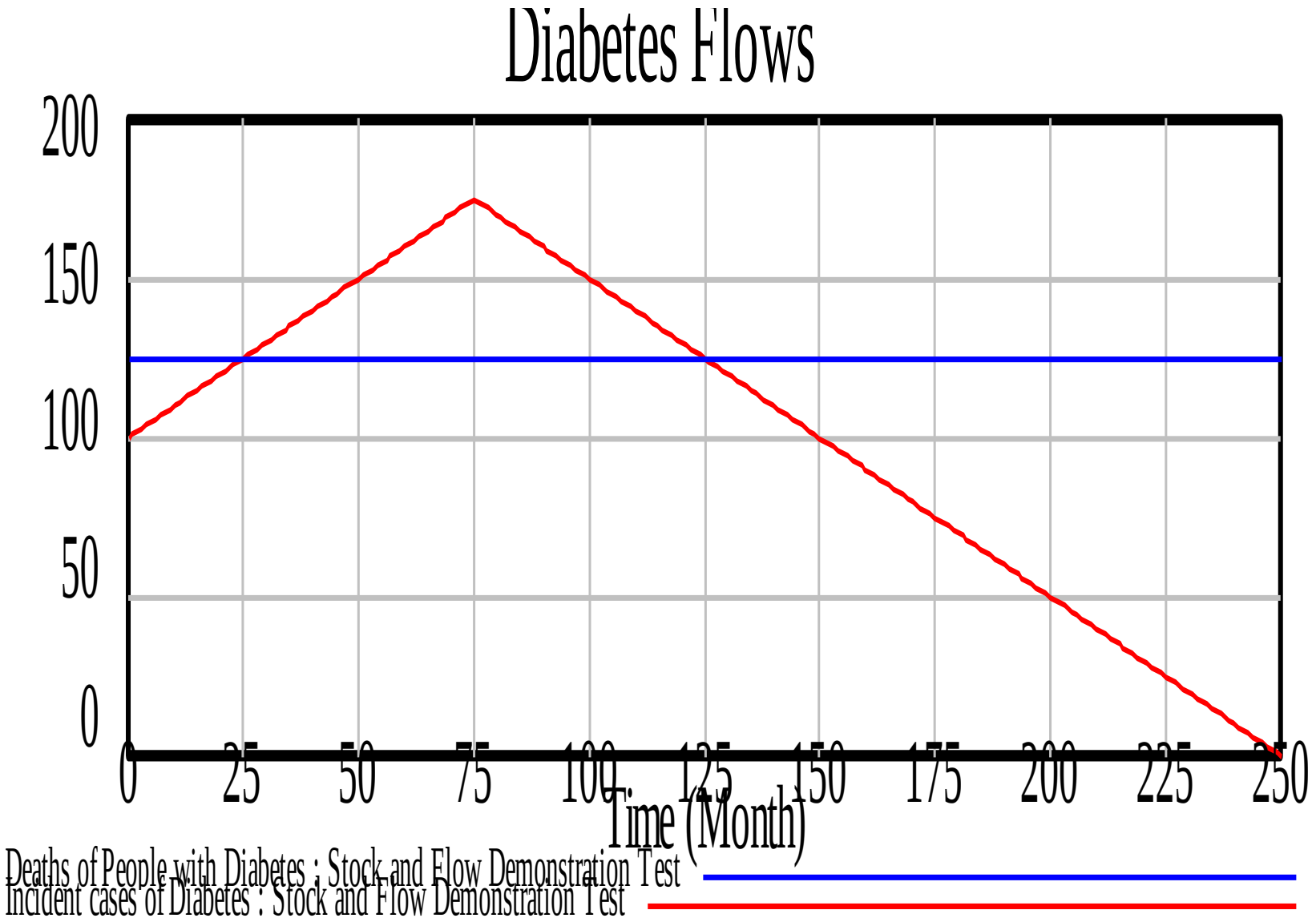


What happens to the stock?

Resulting Stock (Green)

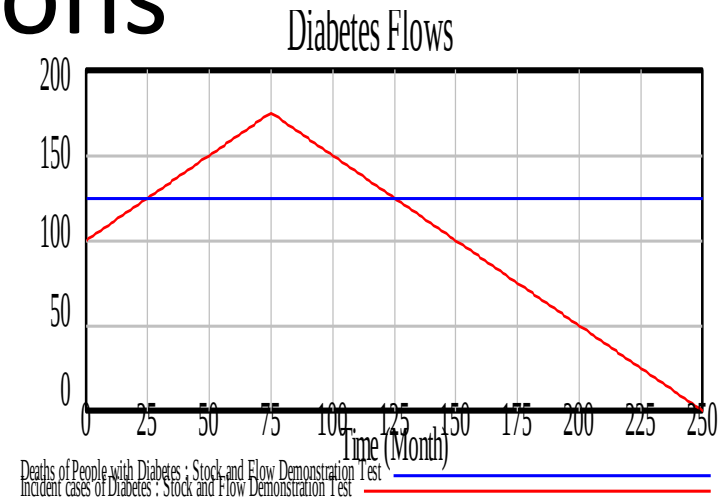


Suppose we have these Flows (Rates)



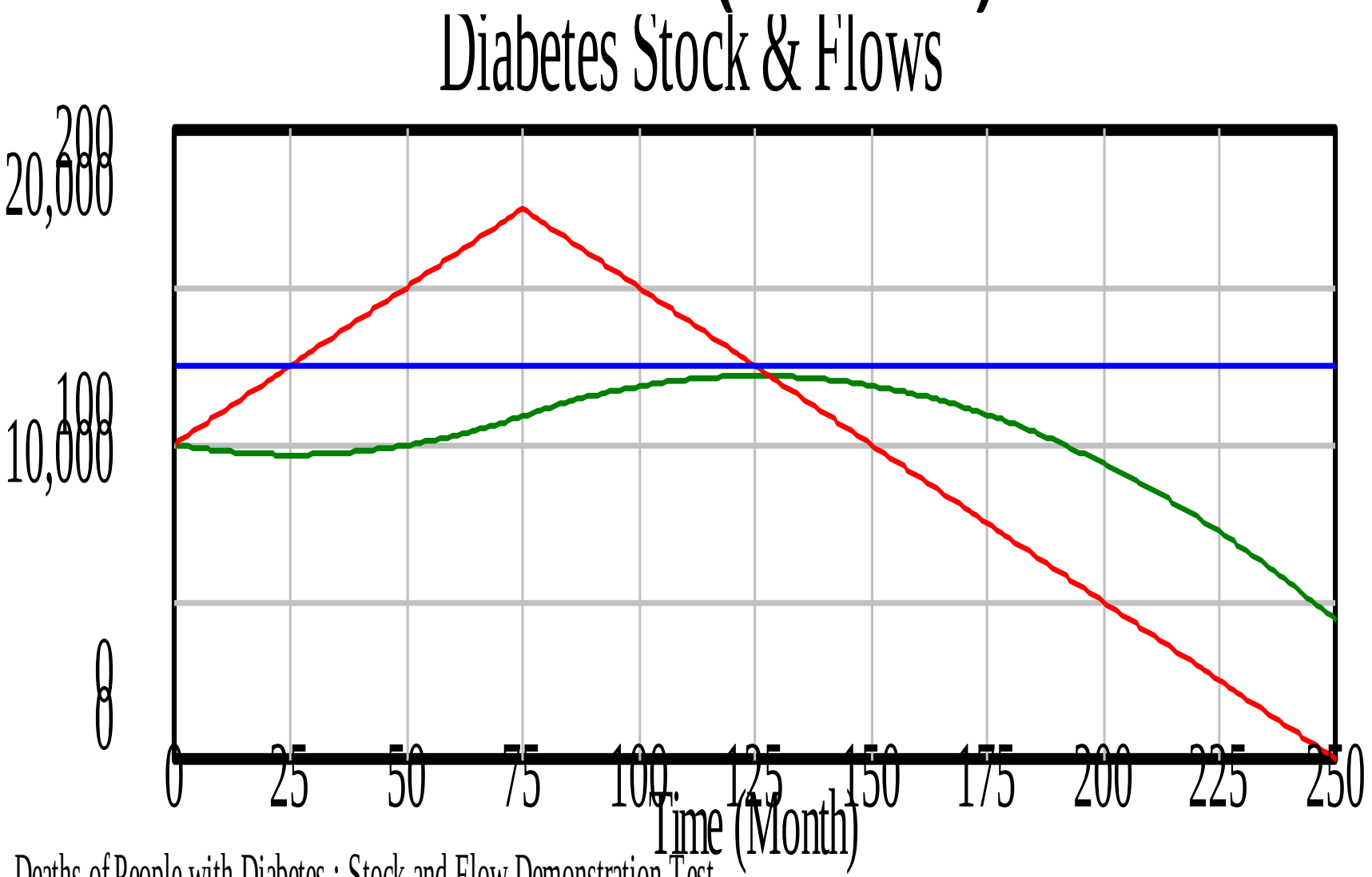
What happens to the stock?

Some Questions



- When is the stock of people with diabetes at its lowest value?
- When is the stock of people with diabetes at its greatest value?
- Is the value of the stock of people with diabetes larger at the beginning or end?
- When is the stock of people with diabetes not changing?

Stock (Green)


Diabetes Stock & Flows
20,000
10,000
0
0
25
50
75
100
125
150
175
200
225
250
Time (Month)
Deaths of People with Diabetes : Stock and Flow Demonstration Test
Incident cases of Diabetes : Stock and Flow Demonstration Test
People with Diabetes : Stock and Flow Demonstration Test

Key Points

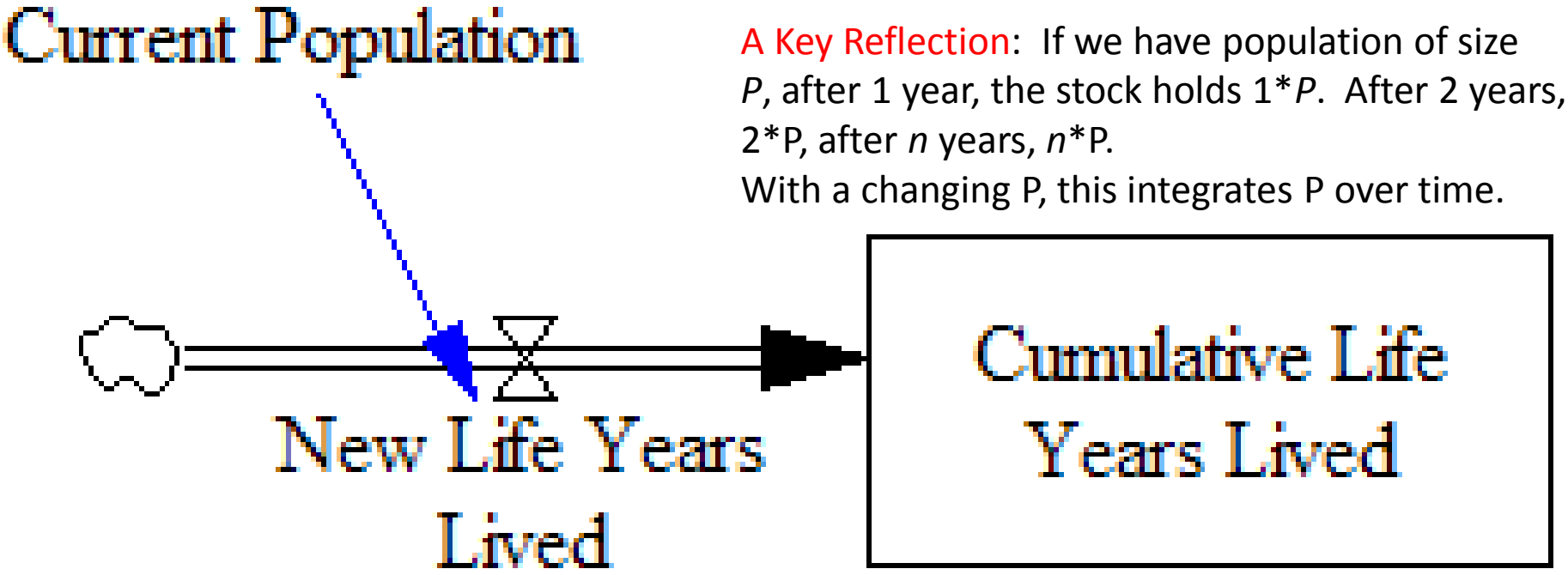
- Maximum value of stock occurs at different time than maximum of flows
  - Example: Maximum prevalence can occur at different time than maximum of incidence

Flows and Feedbacks

- Stocks are always changed by flows
- In your experiments, we've used constant values for flows
- In general, the formulas for the flows will depend on things that are changing (state)
  - Ultimately, these things must depend on the things that collectively specify the state – the stocks!

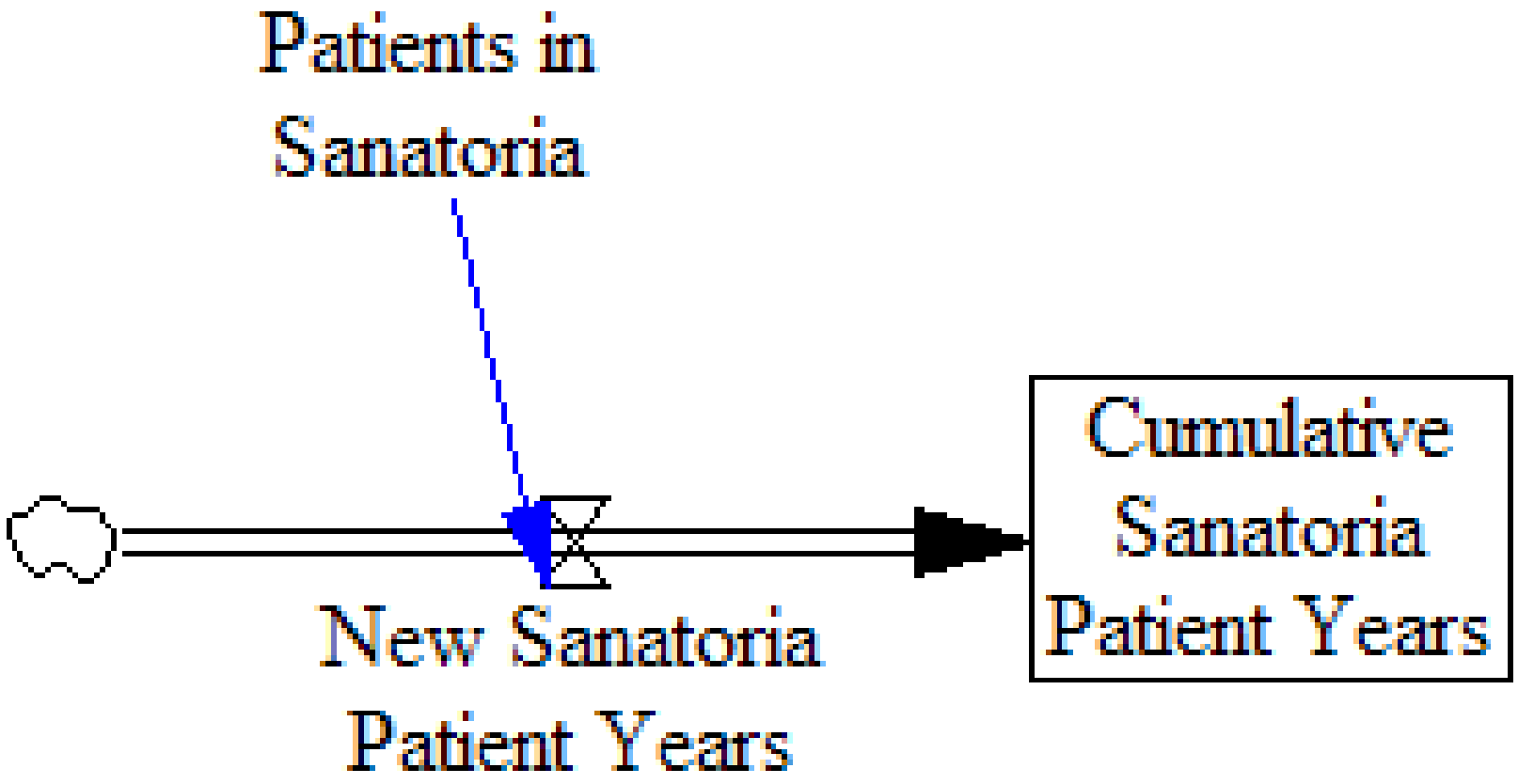
Stocks As Accumulations

- We often use stocks to accumulate (integrate) other (evolving) quantities over time
- Example (assume time measured in years):

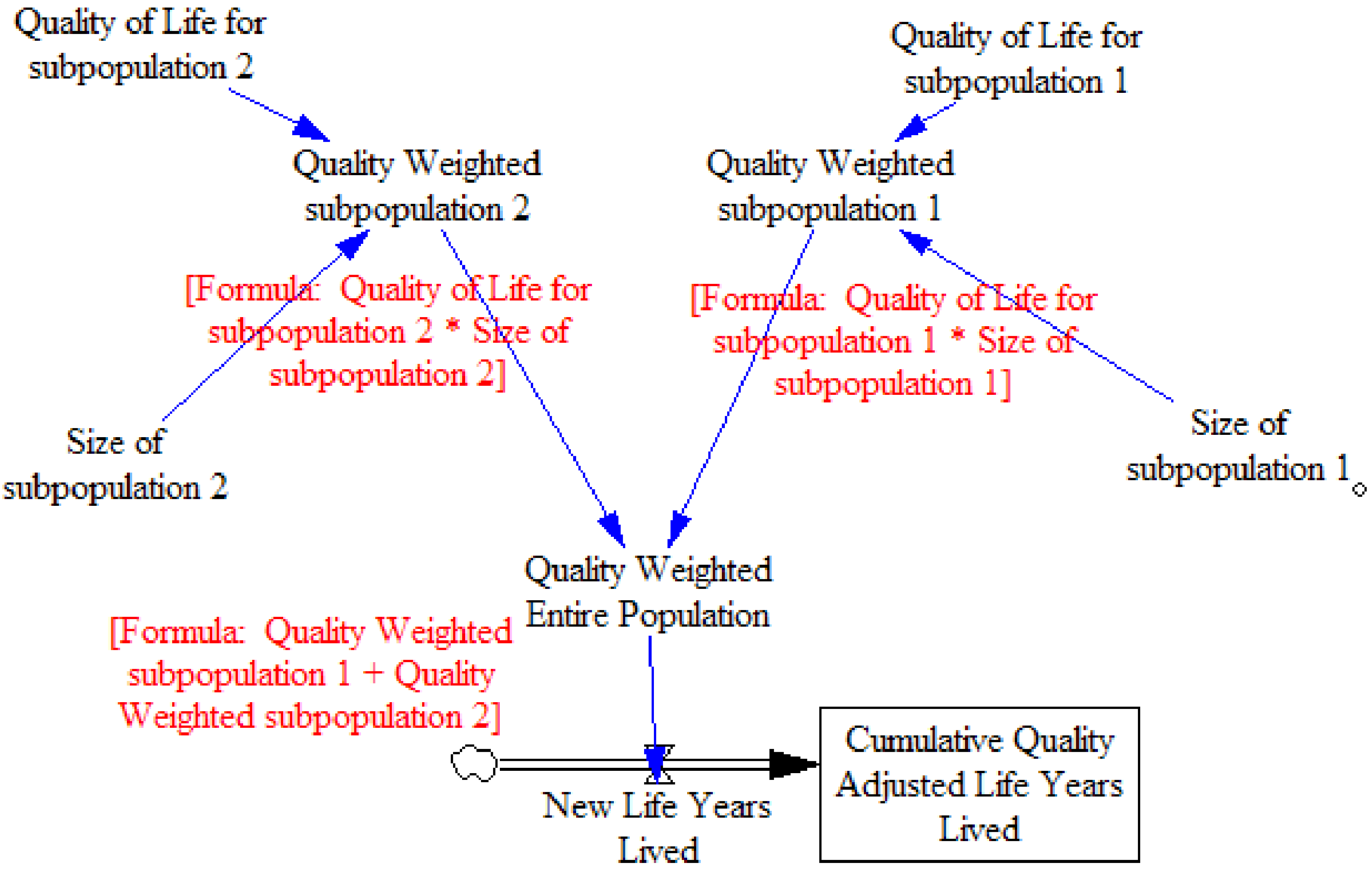


A Key Reflection: If we have population of size *P*, after 1 year, the stock holds 1**P*. After 2 years, 2*P, after *n* years, *n**P.
With a changing P, this integrates P over time.

Example 2


Patients in
Sanatoria
Cumulative
Sanatoria
Patient Years
New Sanatoria
Patient Years

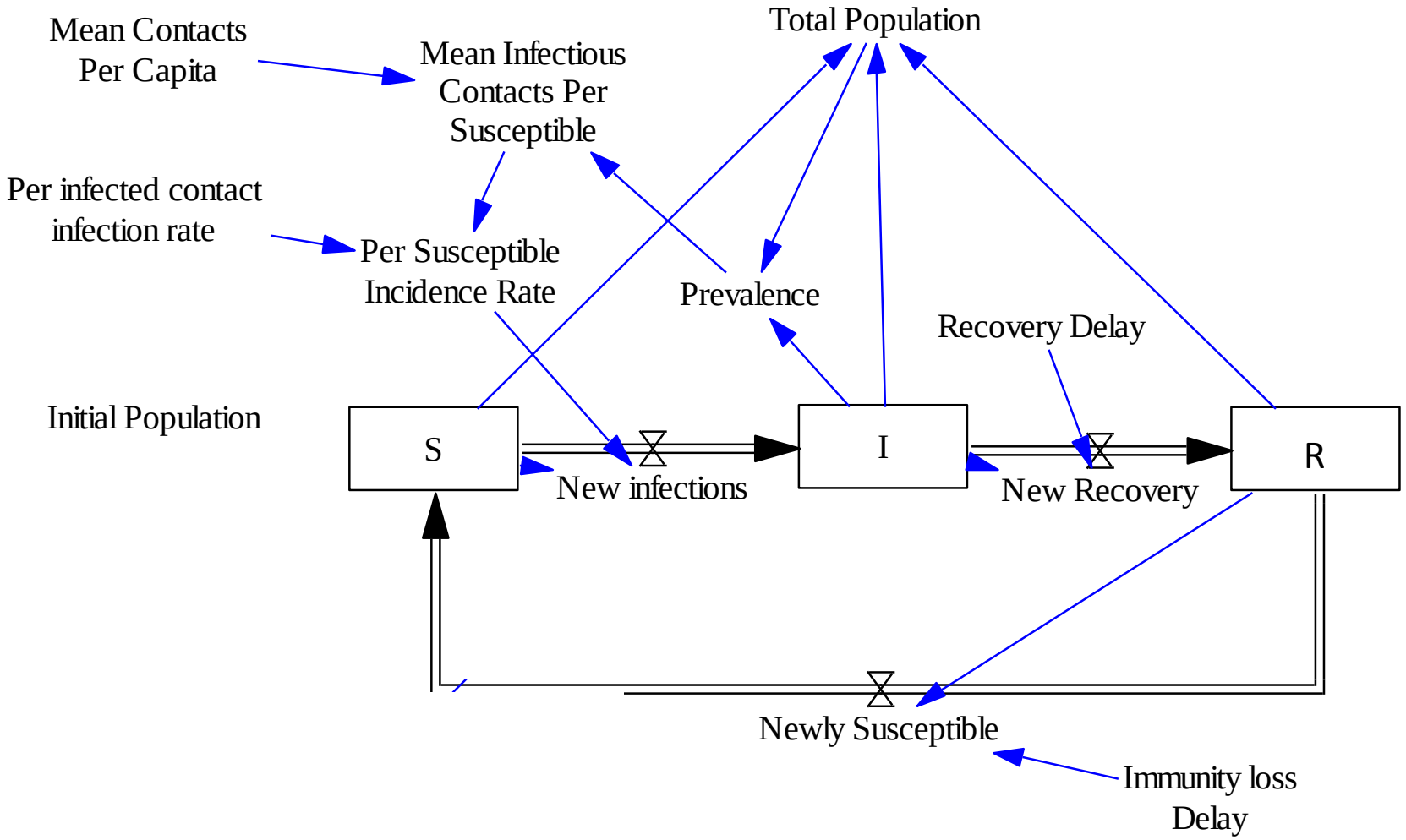
Slightly more Sophisticated



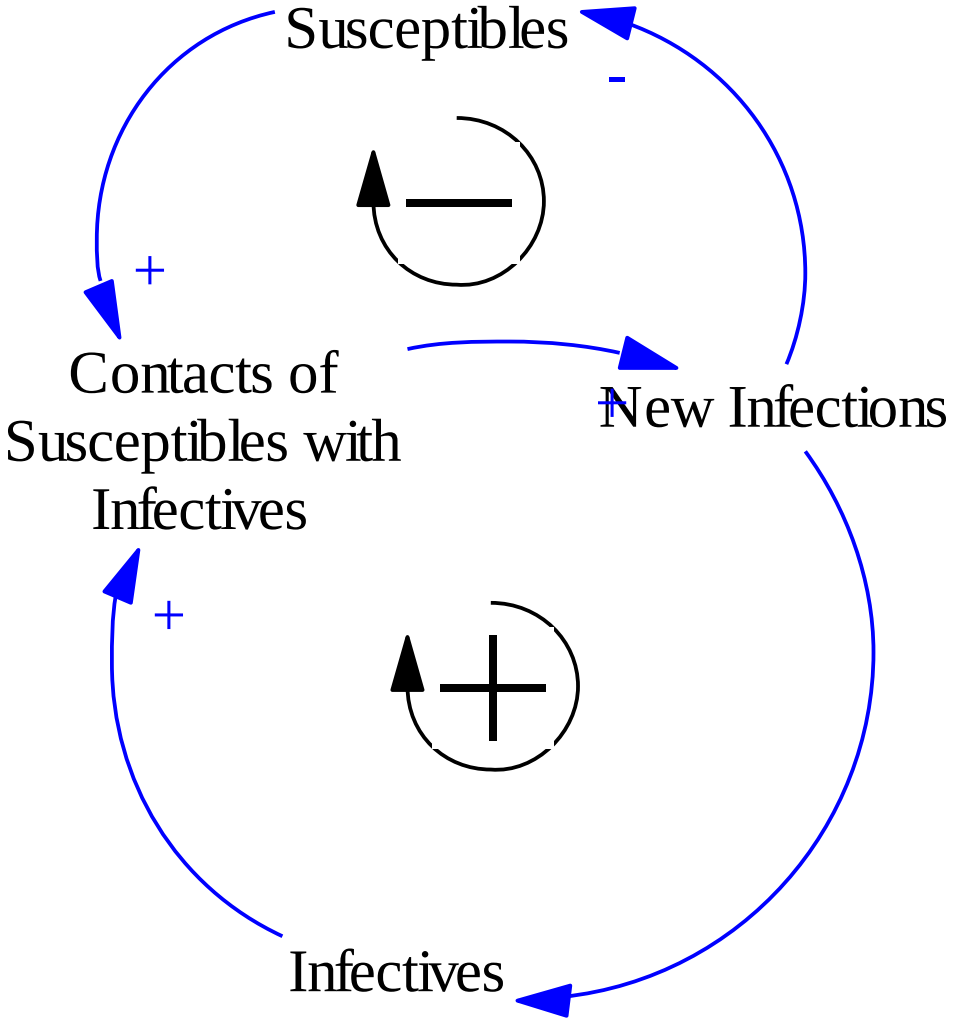
Principle: Structure Determines Behaviour

- Feedback & stock-and-flow structure of a system determines the possible patterns of behaviour
- Different sets of parameters (e.g. values for constants) will select particular behaviour within these behaviour patterns
- Changes to the feedback structure can change behaviour in fundamental ways

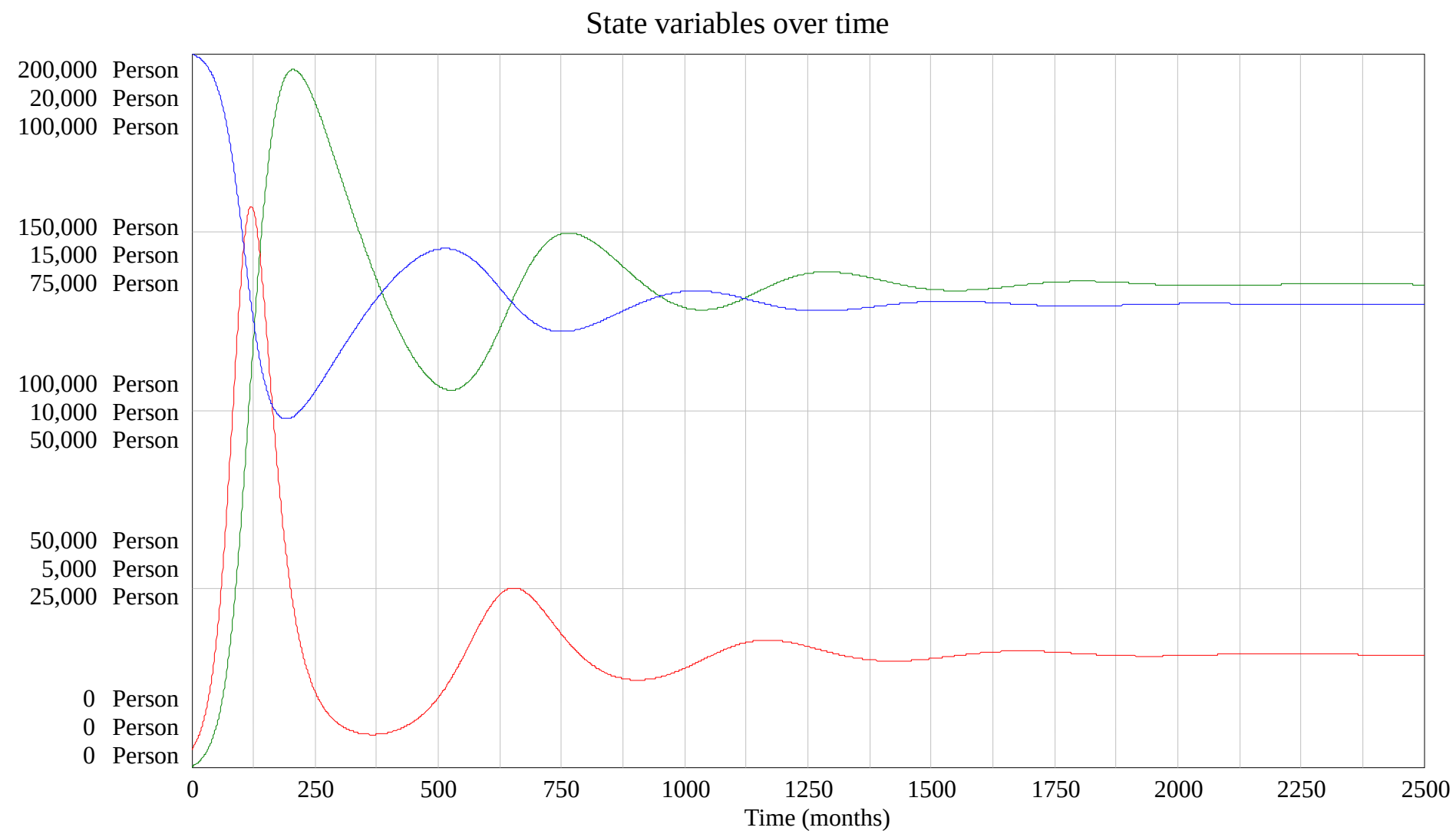
Simple SIT Model



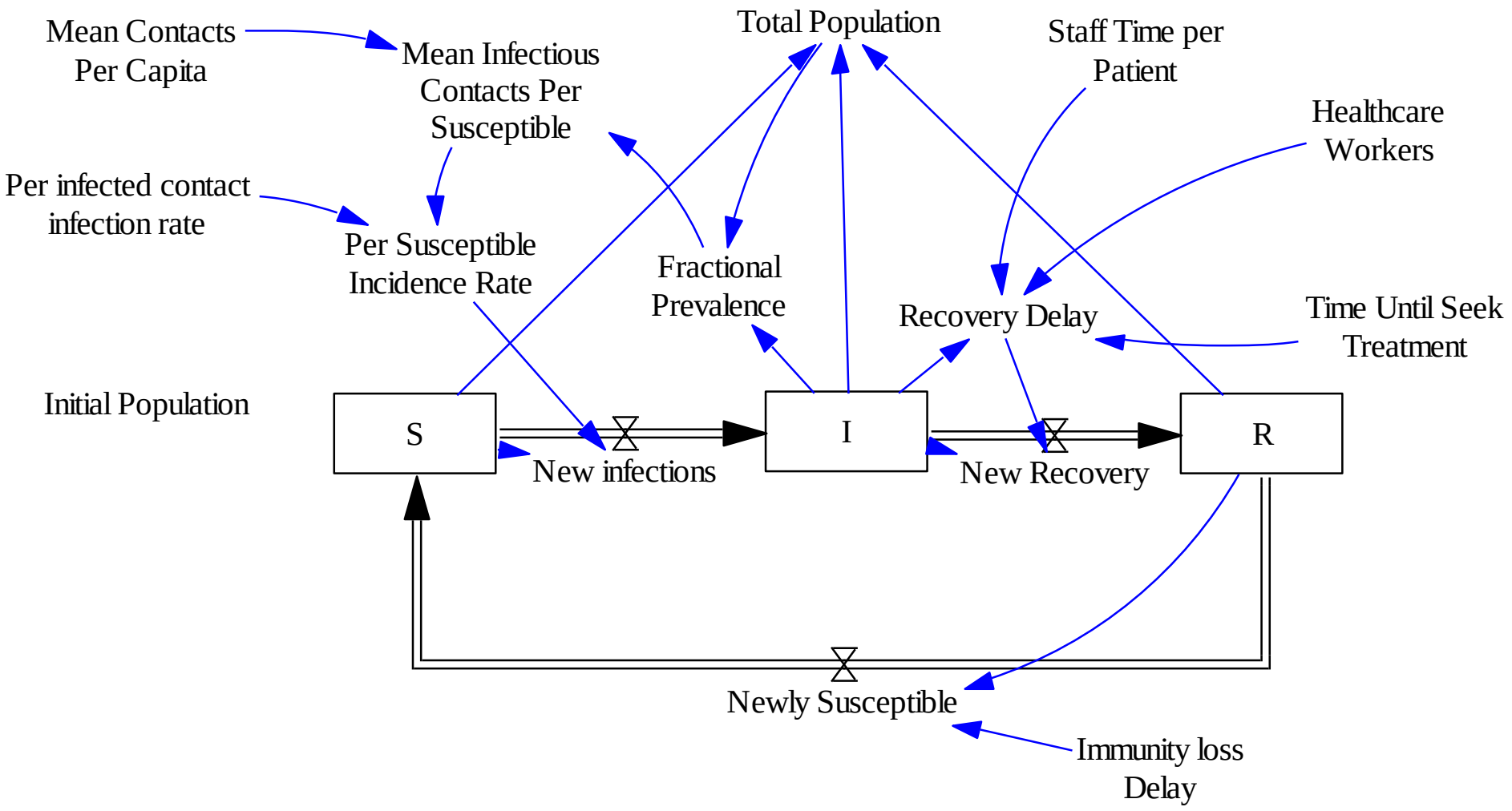
Classic Feedbacks



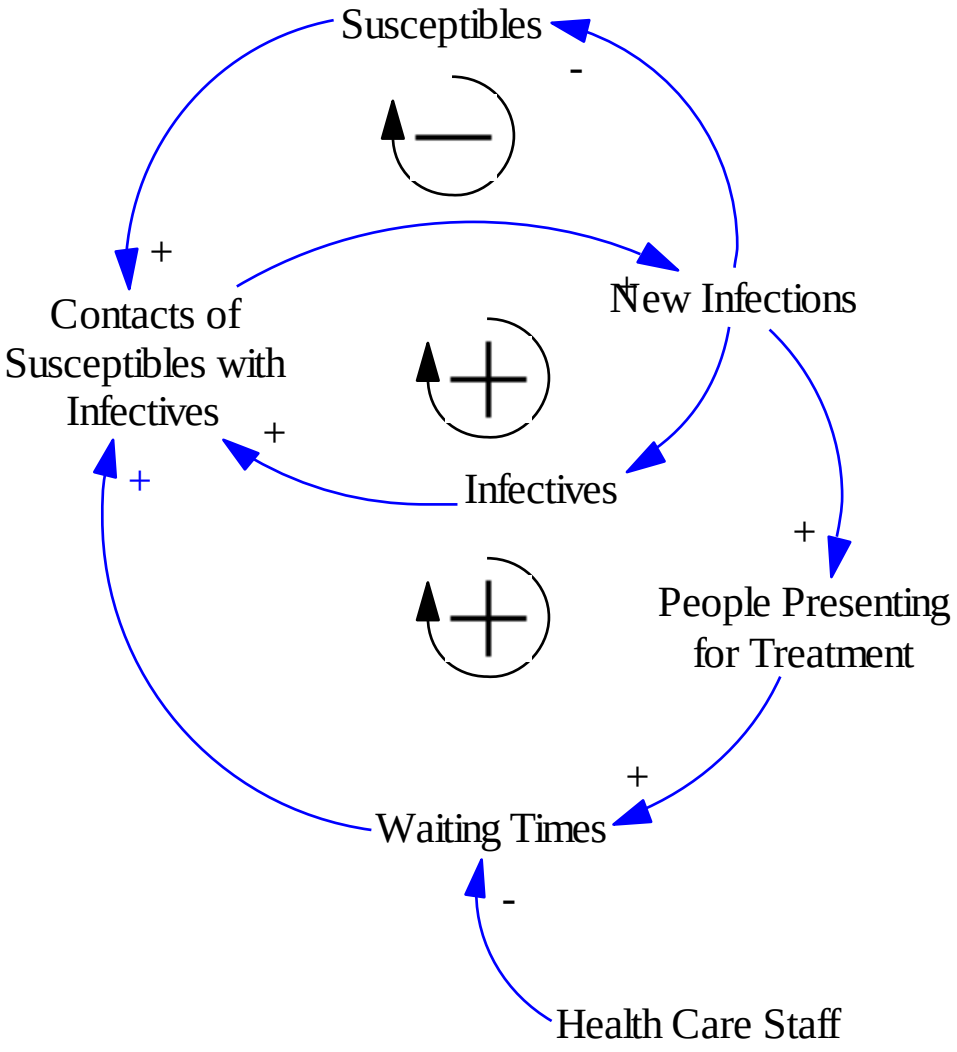
Dynamics


State variables over time
200,000 Person
20,000 Person
100,000 Person
150,000 Person
15,000 Person
75,000 Person
100,000 Person
10,000 Person
50,000 Person
50,000 Person
5,000 Person
25,000 Person
0 Person
0 Person
0 Person
0
250
500
750
1000
1250
1500
1750
2000
2250
2500
Time (months)
S : Alternative 30 HC Workers Exogenous Recovery Delay
Person
I : Alternative 30 HC Workers Exogenous Recovery Delay
Person
R : Alternative 30 HC Workers Exogenous Recovery Delay
Person

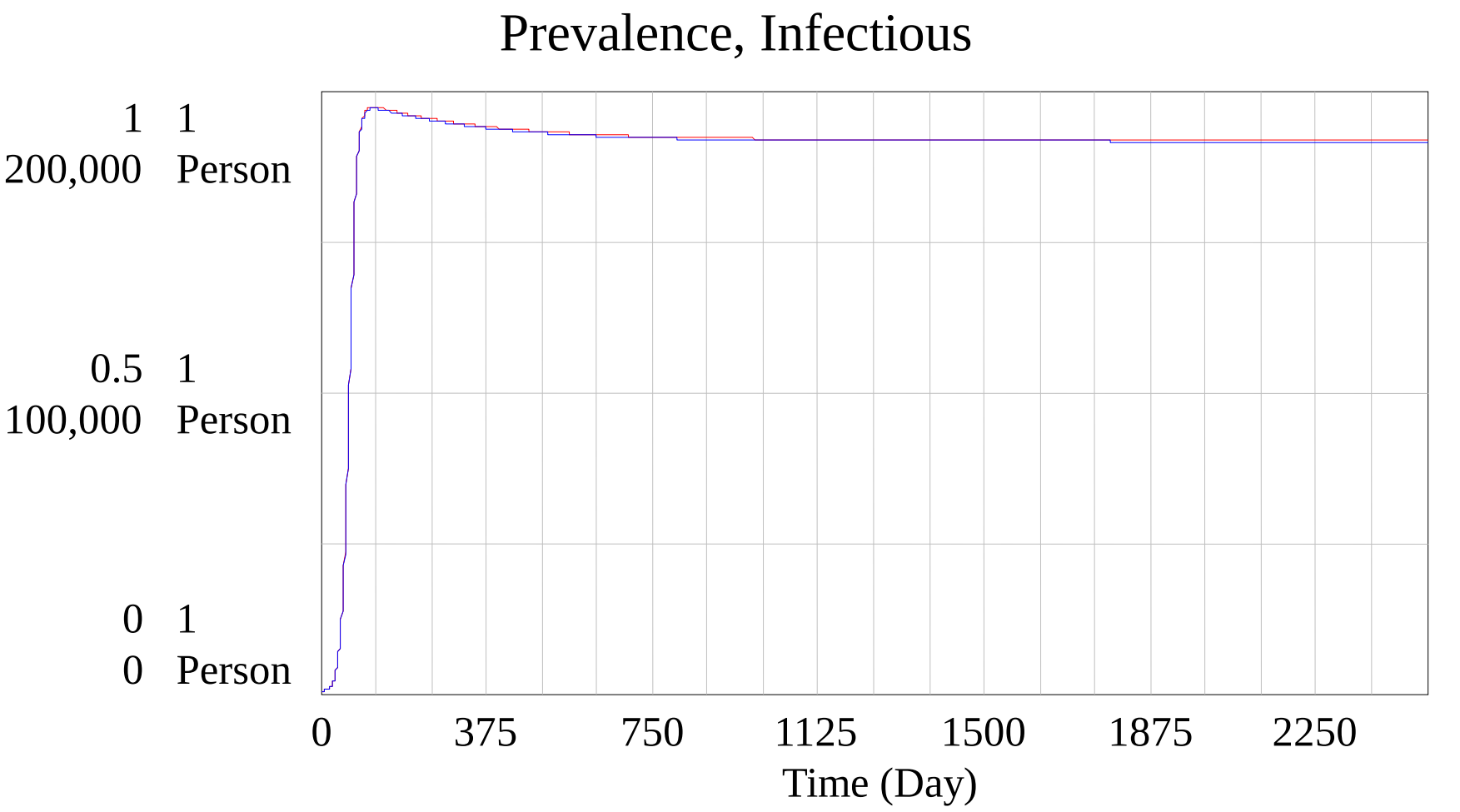
Broadening the Model Boundaries: Endogenous Recovery Delay



Broadening the Model Boundaries: Endogenous Recovery Delay



A Different Behaviour Mode



Structure as Shaping Behaviour

- System structure is defined by
  - Stocks
  - Flows
  - Connections between them
- Nonlinearity: The behaviour of the whole is more than the sum of the behaviour of the parts
  - “Emergent” behaviour would not be anticipated from simple behaviour of each piece in turn
- Stock and flow structure (including feedbacks) of a system determines the qualitative behaviour modes that the system can take on